Physics 221

Department of Physics
The Citadel

Lecture Notes

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Rotational Equilibrium

Announcements

- Homework Set 12: due Wednesday

  Sections 11.5 and 12.1 – 12.3.

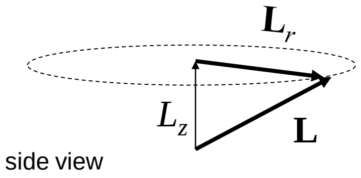
  Problems: Two on torque & angular momentum vectors, plus Ch. 12: 9, 11, 13, 14, 45, 59.

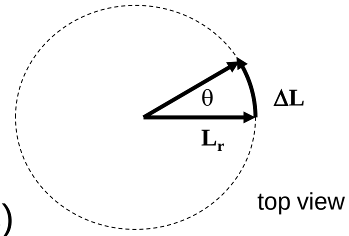
Read sec. 1 – 3 in Chapter 12 only.

Exam 3: Chapter 9 – 12.3, except 9.7, 9.8.
next Monday.
Probably 3 questions.

Hint on Problems 1 & 2

Both problems 1 and 2 have an angular momentum vector that rotates on a cone, as for the conical pendulum. The geometry is like this, with L_z conserved and L, L_r constant, but rotating. (L_r means “radial component”.)





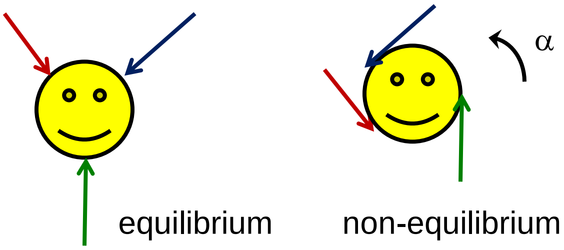
$\Delta\mathbf{L} = \boldsymbol{\tau}\,\Delta t$, $|\Delta\mathbf{L}| = L_r\,\theta$ (radians)

Static Equilibrium

- In static equilibrium, $\mathbf{F} = 0$.
- But also, there can be no rotational acceleration, so $\tau = 0$.
- These conditions apply to no motion, or constant motion ($\mathbf{v}$, ω).

Example:

same forces applied differently

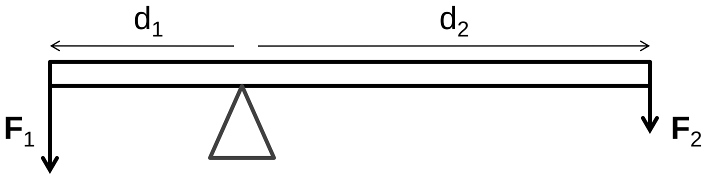


Lever Law

- The simplest case is the Lever Law (Archimedes): “Magnitudes are in equilibrium at distances proportional to their weight.”

In modern language, this means that torques balance about the pivot:

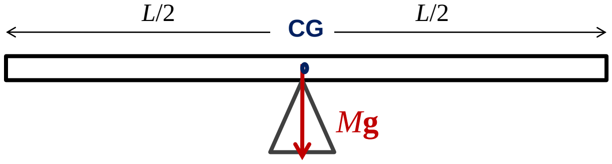
$$F_1 d_1 = F_2 d_2.$$



Center of Gravity.

A beam balances at its center of gravity. This is the point at which all the torques caused by gravity acting all along the beam balance. In a uniform gravitational field, the center of gravity is also the center of mass.

The gravitational force can be considered to be $M\mathbf{g}$ downward **at** the center of gravity.



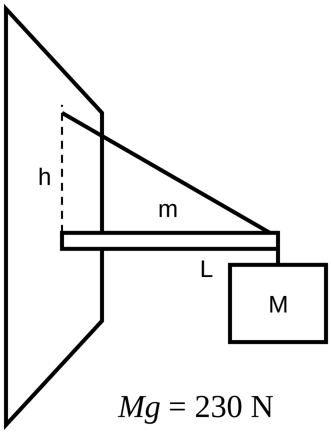
Sign Problem

A sign weighing 230 N is supported by a 90 N beam of length 1.75 m.

The guy wire is attached to a point 1.20 m above the beam.

Find the tension in the guy wire and the force of the wall on the beam.

Strategy: Balance forces and torque on the beam.



Mg = 230 N
mg = 90 N
L = 1.75 m
h = 1.20 m

Sign Problem

Mg = 230 N
mg = 90 N
L = 1.75 m
h = 1.20 m

Balance torque about left end of beam:

$\tau = LT \sin\theta - MgL - mgL/2$

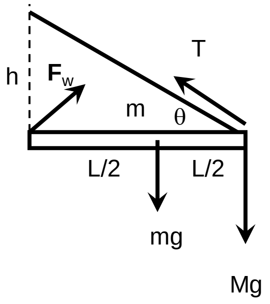
$= 0.$

$T = (M + m/2)g / \sin\theta$

$\tan\theta = h/L = 0.6857,$

$\theta = 34.4^{o}, \sin\theta = 0.565.$

T = 275 N / 0.565 = 487 N.



Sign Problem

$Mg = 230$ N
$mg = 90$ N
$L = 1.75$ m
$h = 1.20$ m
$T = 487$ N

$F_{wx} = T\cos\theta = 402$ N.

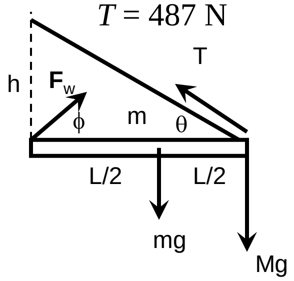
Balance torque about the right end of the beam:

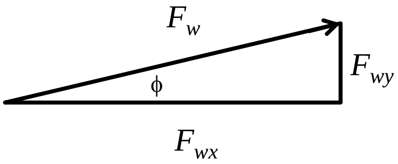
$LF_{wy} = Lmg/2$.

$F_{wy} = mg/2 = 45$ N.

$$F_w = \sqrt{F_{wx}^2 + F_{wy}^2} = 405 \text{ N},$$

$$\phi = \tan^{-1}\left(\frac{F_{wy}}{F_{wx}}\right) = 6.39^{\circ}.$$

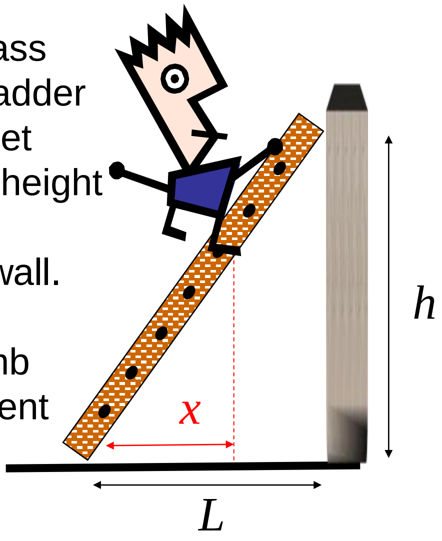




Slippery Ladder

Suppose a person with a mass $M = 90$ kg wants to climb a ladder with mass $m = 15$ kg that is set against a frictionless wall of height $h = 3.0$ m, with the bottom a distance $L = 1.5$ m from the wall.

How far can the person climb before slipping if the coefficient of friction against the floor is $\mu_s = 0.20$?



Slippery Ladder

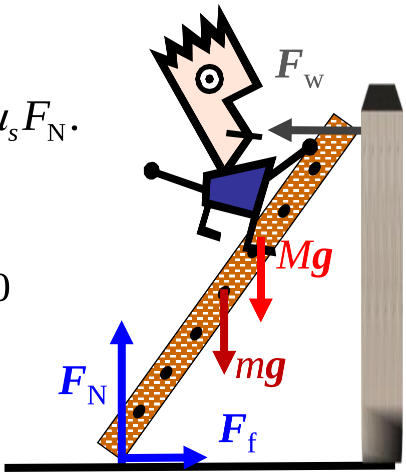
Static friction limit: $F_\mathrm{f} \le \mu_s F_\mathrm{N}$.
Balancing forces:

$$x\!: \quad F_\mathrm{f} - F_\mathrm{w} = 0$$

$$y\!: \quad F_\mathrm{N} - mg - Mg = 0$$

Combine:

$$F_\mathrm{w} = F_\mathrm{f} \le \mu_s (M + m) g$$
$$= 206 \text{ N.}$$



Slippery Ladder

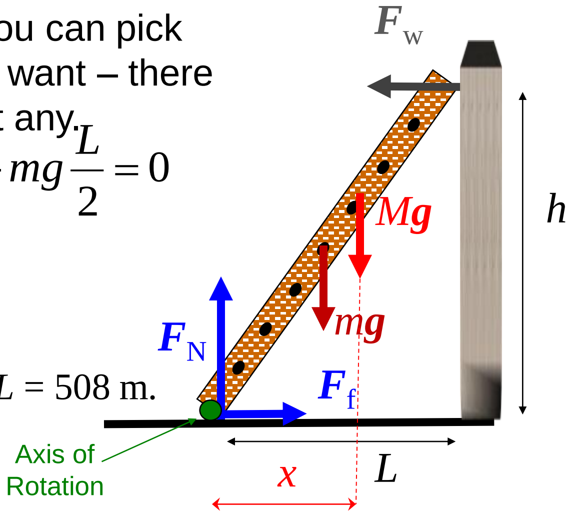
Torque equation: you can pick the axis where you want – there is no rotation about any.

$$\sum \tau = F_\mathrm{w} h - Mgx - mg\frac{L}{2} = 0$$

$$Mgx = F_\mathrm{w} h - mg\frac{L}{2}$$

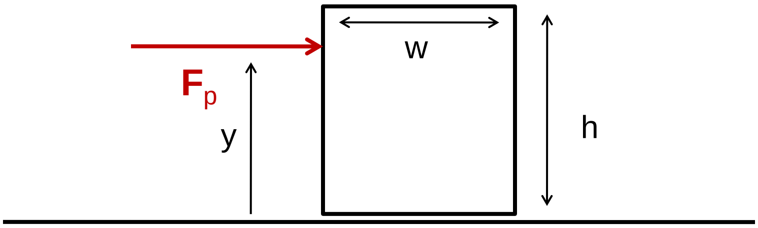
Numerically,
$882x = 206\,h - 73.5\,L = 508$ m.

$x = 0.576$ m.



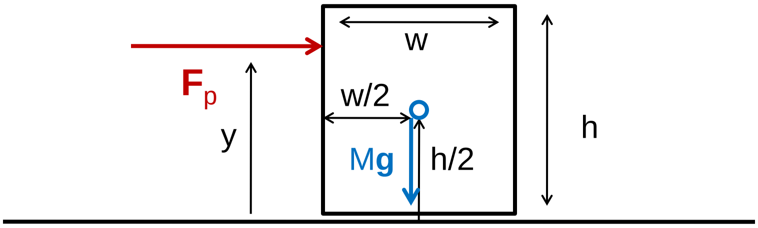
Pushing A Box

If I want to push a uniform box across the floor at a constant speed, what is the highest I can push it horizontally without tipping it? Assume the coefficient of friction is μ.



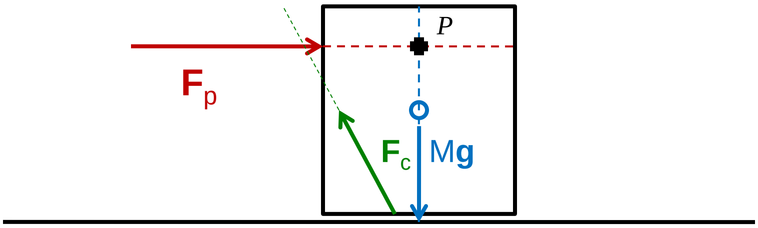
Pushing A Box

All the forces and torques must balance just before it tips, but where do they act? The weight of the box acts at the CM. But what about the contact force: the vector sum of friction and the normal force, $\mathbf{F}_c = \mathbf{F}_f + \mathbf{F}_N$? It acts somewhere on the bottom, but where should we put it?



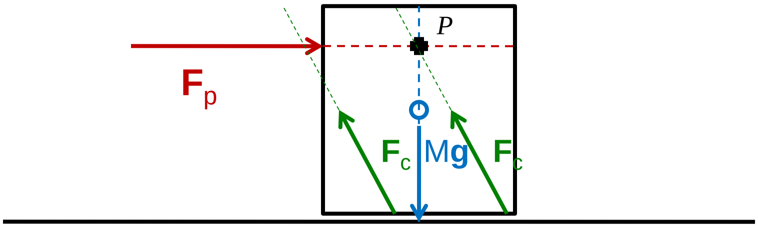
Pushing A Box

In general, if the box is moving at a constant speed and not tipping, the torques balance about any axis, in particular one through the intersection P of $\mathbf{F}_\mathrm{p}$ and $M\mathbf{g}$. The only force that could produce a torque about P would be the contact force.



Pushing A Box

The torque can be made to go away by sliding it until the three lines of force intersect at P. Then there is no torque about P. This always works to balance the torques of three non-collinear forces in an inertial reference frame.



Pushing A Box

This identifies the point where the contact force acts: a distance x in front of the CM such that

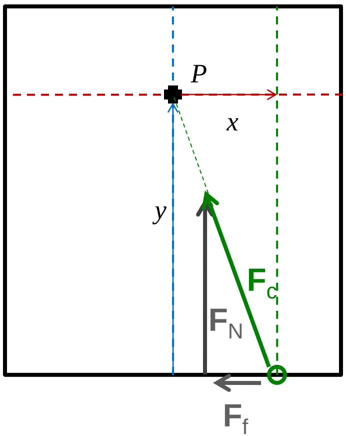
$x/y = F_f / F_N = \mu$,

by similar triangles.

The furthest forward the Contact force can act is At the front corner:

$x = w/2$

Then $y = x/\mu = ½\, w/\mu$.



Pushing A Box

A simpler solution, starting from the reasonable assumption that the normal force acts on the front corner when the box is just about to tip. Used at 8AM only.

You can get the same result by assuming that only the front corner is in contact with the ground when it is about to tip.

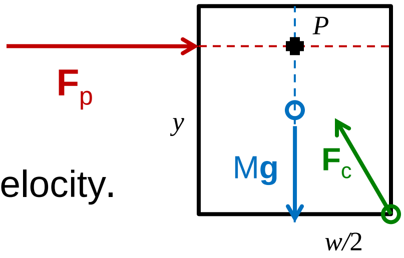
If you balance torque about that point, the contact force doesn't enter at all.

$Mg\, w/2 - F_p y = 0$.

Then $y = ½\, Mgw/F_p$,

with $F_p = \mu Mg$ at constant velocity.

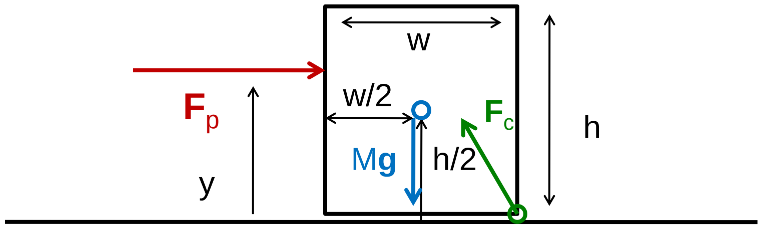
This again gives $y = ½\, w/\mu$.



Accelerating Box

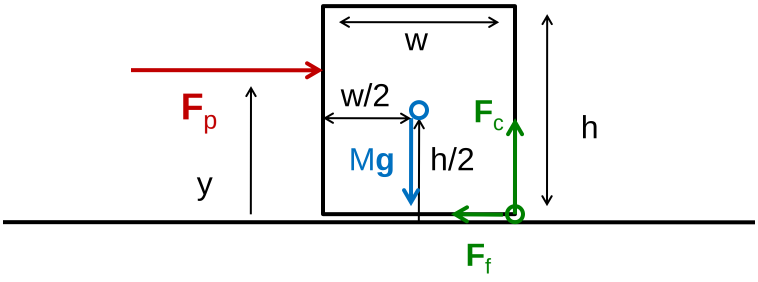
This example was not used in class, but serves as a warning that you are **not** free to choose any rotation axis if the object is accelerating.

What is the condition for the box to be on the verge of tipping if it is accelerating at rate a?



Accelerating Box

Now the only point about which the sum of torques vanishes is the CM. We can still assume only the front edge is in contact with the ground, so $\mathbf{F}_f$ and $\mathbf{F}_N$ act there. $\mathbf{F}_c$ and $\mathbf{F}_p$ must produce opposite torques about the CM.



Accelerating Box

Balancing torques about the CM gives

$$F_p(y - \tfrac{1}{2}h) = \tfrac{1}{2}Mgw - \tfrac{1}{2}\mu Mgh.$$

Newton’s Law implies that $F_p = Ma + \mu Mg$.

$$M(a + \mu g)(y - \tfrac{1}{2}h) = \tfrac{1}{2}Mgw - \tfrac{1}{2}\mu Mgh$$

$$(a + \mu g)y = \tfrac{1}{2}gw - \tfrac{1}{2}\mu gh + \tfrac{1}{2}h(a + \mu g)$$

$$y = \frac{gw + a\,h}{2(\mu g + a)}$$

$a = 0$ gives the previous result, $y = \tfrac{1}{2}w/\mu$.

For very large a, $y \rightarrow h/2$.

Accelerating Box

When the box is about to tip, the forces look like this. $\mathbf{F}_p$ and $\mathbf{F}_c$ produce opposite torques about the CM. The three forces **do not** intersect at a point when the box is accelerating. That is only true when the box is at rest or moving at a constant velocity.