Physics 221

Sections 1 and 2

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Motion in One Dimension

Part 2: Accelerated Motion

Announcements

- Answers have been posted for problem set 1. Complete solutions will appear soon (Wed.): look for a link to a PDF file in the problem set.
- Set 2 is open and due Wednesday. It contains problems 3, 5, 9, 13, 21, 23, 39, and 44 from Chapter 2.
- Next time, we will discuss problem set 2 and you will work some in-class problems in groups (ideally, 2 per group, 3 at most). You may wish to bring your **calculator and textbook**, but I will write the essential equations on the board.

Today’s Topics

Today, we will

- discuss accelerated motion: what happens when the velocity changes.
- look in more detail at instantaneous velocity.
- define instantaneous and average acceleration.
- consider the special case of constant acceleration.

Instantaneous Velocity

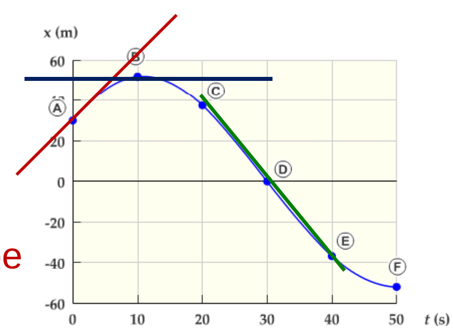
The instantaneous velocity is

$$v_x = dx/dt.$$

The maximum velocity occurs where the slope is the greatest:

point A. (sign matters)

The minimum velocity occurs where the slope is most negative: point D.



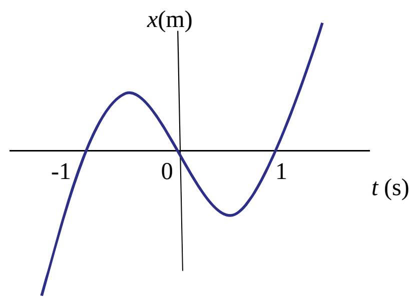
The velocity is zero at a turning point (B), where the slope is zero.

Example

Suppose that with x measured in meters and t in seconds,

$x = t^3 - t.$

What is $v_x(t)$?



The only derivative we will need in this chapter is

$$\frac{dt^n}{dt} = nt^{n-1}$$

(n can be any number.)

Also, the derivative has the properties

$$\frac{d(ax)}{dt} = a\frac{dx}{dt}, \quad \frac{d(x+y)}{dt} = \frac{dx}{dt} + \frac{dy}{dt}.$$

Example

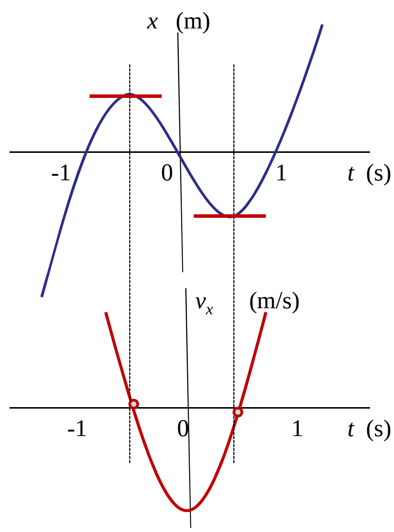
Taking derivatives is a linear operation, so this applies term by term:

$v_x(t) = dx/dt$

$= 3\,t^2 - 1.$ [in m/s]

Note that $v_x(t) = 0$ at $t = \sqrt{3}$ s or $-\sqrt{3}$ s where the particle turns around.

The velocity is always zero at a turning point.

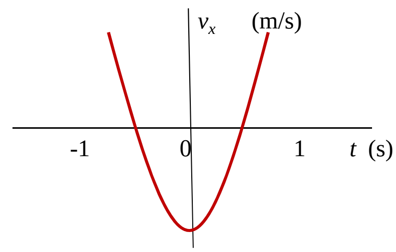


Acceleration

When we have a graph of velocity vs time, we can go a step further and ask how the velocity changes.

The change in velocity per unit time is called **acceleration**.

The acceleration is the slope of a line tangent to the curve $v(t)$.



In calculus terms, acceleration is the derivative of velocity with respect to time:

$$a_x = \frac{dv_x}{dt}$$

Example

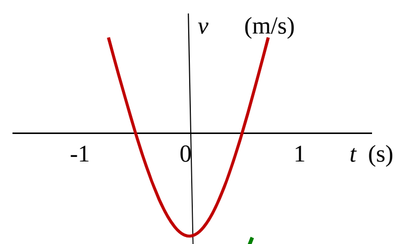
We found that if $x(t) = x^3 - x$, then

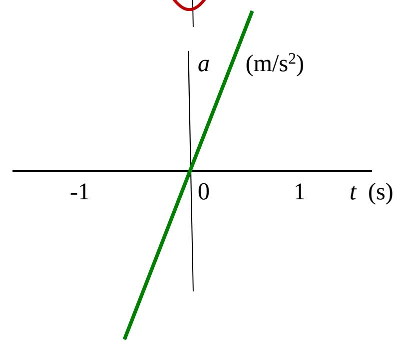
$v_x(t) = dx/dt$

$= 3\,t^2 - 1.$ [in m/s]

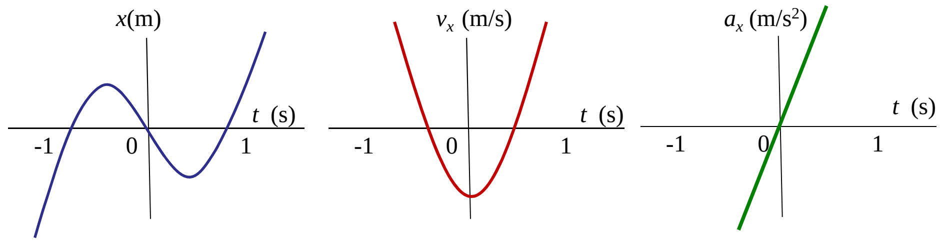
What is the acceleration?

$a_x(t) = 6t$ [in m/s^2].





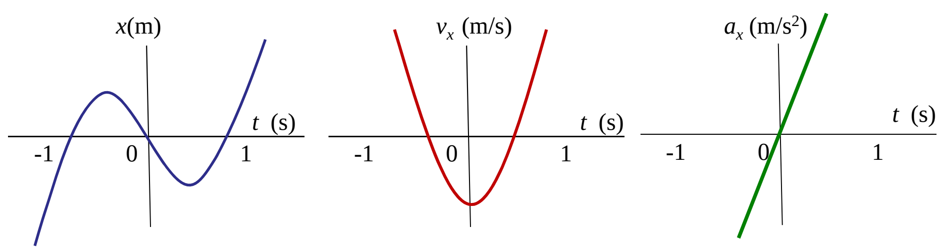
Second Derivatives



If v_x is the derivative of x, and a_x is the derivative of v_x, then a_x is the **second derivative** of x with respect to time.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

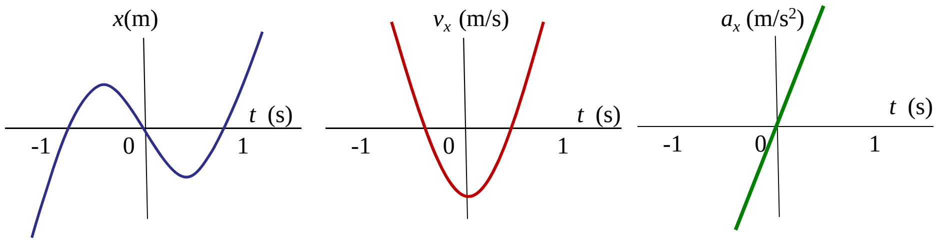
Second Derivatives



- v_x is positive when x is increasing, negative when x is decreasing, and zero at turning points.
- a_x is positive when v_x is increasing, negative when v_x is decreasing, and zero when it is momentarily constant.

- Can we say anything graphical about how the acceleration relates to the position graph?

Second Derivatives



- a_x is positive when the slope of $x(t)$ is increasing, and negative when the slope is decreasing.
- This means that is positive where $x(t)$ is concave upward, and negative where $x(t)$ is concave upward.
- The point in between ($t = 0$ in the example) is called an **inflection point**. Here, the acceleration is zero.

Average Acceleration

We can also define the average acceleration for a trip where the acceleration is changing, in analogy with average velocity.

The average acceleration is $a_x^{\text{avg}} = \dfrac{\Delta v_x}{\Delta t}$.

If a car goes from zero to 60 mi/hr in 5.0 seconds, what is its average acceleration in m/s^2?

$$a_x^{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{\left(60\,\frac{\text{mi}}{\text{hr}}\right)\left(\frac{1000\text{m}}{0.62\text{mi}}\right)\left(\frac{1\text{hr}}{3600\text{s}}\right)}{5.0\text{s}} = 5.4\,\frac{m}{s^2}$$

Free Fall

- Another example of constant acceleration is falling objects.
- Galileo demonstrated that objects fall to earth with the same acceleration, regardless of size or composition, if air resistance can be neglected.
- On or near Earth, the gravitational acceleration is $g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2$ downward.
- This applies whether the object has been thrown or dropped, as long as only gravity acts on it.

Average Acceleration

- How many meters does the car travel during this 5.0 second acceleration?

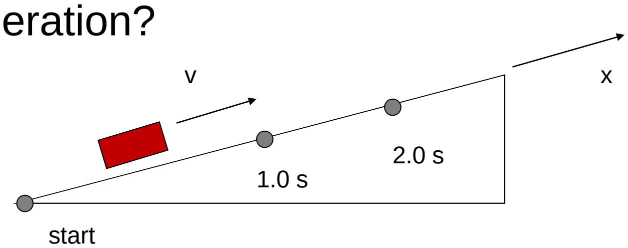
We know the $\Delta x = v_x^{\text{avg}}\, t$. If the velocity is increasing at a constant rate from 0 to 60 mph, then $v_x^{\text{avg}} = 30$ mph.

$$\Delta x = \left(30\,\frac{\text{mi}}{\text{hr}}\right)(5.0\ \text{s})\left(\frac{1\ \text{hr}}{3600\text{s}}\right)\left(\frac{1000\ \text{m}}{0.62\ \text{mi}}\right) = 67\ \text{m.}$$

Toy Car

A child's toy car rolling across a sloping floor is known to have constant acceleration.

Taking $x = 0$ at $t = 0$, it is observed that the car is at $x = 2.66$ m at $t = 1.0$ s and $x = 4.25$ m at $t = 2.0$ s. What is the car's acceleration?



Toy Car

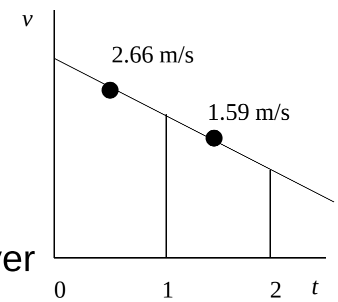
If we know the positions and times, we can find the average velocities in the intervals from $0 - 1$ s and from $1 - 2$ s:

$v_{01} = 2.66$ m/s, $v_{12} = 1.59$ m/s.

How can this information be used to find the acceleration?

If we know the instantaneous velocities at two points, we can calculate $a = \Delta v / \Delta t$.

Toy Car



This is a constant acceleration problem with decreasing velocity, so the velocity decreases linearly with time.

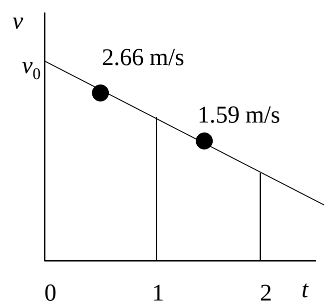
In a linear graph, the average over any interval is the value at the center of that interval.

Thus, $v(0.5\text{ s}) = v_{01} = 2.66\text{ m/s}$,

$v(1.5\text{ s}) = v_{12} = 1.59\text{ m/s}$.

$a = (2.66\text{ m/s} - 1.59\text{ m/s}) / 1\text{ s} = -1.07\text{ m/s}^2$.

Toy Car



- What is the car's initial velocity?

$v - v_0 = at$.

Putting in numbers gives

$2.66\text{ m/s} - v_0$

$= (-1.07\text{ m/s}^2)(0.5\text{ s})$.

$v_0 = 2.66\text{ m/s} + 0.54\text{ m/s}$

$= 3.20\text{ m/s}$.

Constant Acceleration

We've used these equations so far:

$$v_f = v_0 + at$$

$$v_{avg} = ½ (v_0 + v_f)$$

Combine these with $\Delta x = v_{avg}\, t$ to get

$$\Delta x = ½ (v_0 + v_f)\; t$$

$$= ½ (2v_0 + at)\; t = v_0 t + ½ at^2$$

$$\boxed{x = x_0 + v_0 t + ½ at^2}$$

$\Delta x = x - x_0$

Constant Acceleration

Another equation for constant acceleration:

$$v_f^2 - v_0^2 = (v_f - v_0)(v_f + v_0)$$

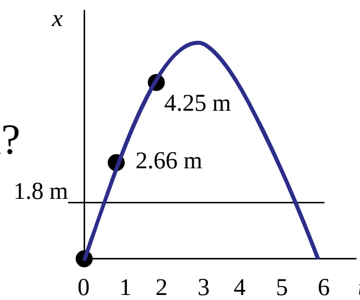
$$= (at)\,(2v_{avg}) = 2a\Delta x.$$

Or

$$\boxed{v_f^2 = v_0^2 + 2a\Delta x.}$$

This is useful when no time information is given.

Toy Car

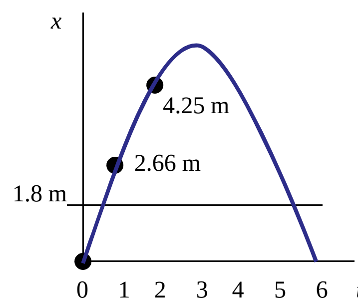


What is the car's velocity when it reaches $x = 1.8$ m?

Use $v_f^2 = v_0^2 + 2a\Delta x$ to find the velocity at 1.8 m:

$$v_f = \pm\sqrt{v_0^2 + 2a\Delta x}$$
$$= \pm\sqrt{(3.2)^2 + 2\times(-1.07)\times(1.8)}\,\frac{\text{m}}{\text{s}} = \pm 2.527\,\frac{\text{m}}{\text{s}}.$$

Toy Car



At what time does the car reach the point $x = 1.8$ m?

Use the previous result together with $\Delta v = at$.

$\Delta v = v_f - v_0 =$

$= (2.527 - 3.200)$ m/s $= -0.673$ m/s

or $(-2.527 - 3.200)$ m/s $= -5.727$ m/s.

$t = \Delta v/a = 0.63$ or 5.35 s.