

Physics 221

Department of Physics
The Citadel

Lecture Notes

S. Yost
October 30, 2009

Introduction to Angular Momentum Fixed Axis

Announcements

- Homework Set 11: due Wednesday
Sections 10.9 and 11.1 – 11.3.
Problems: Ch. 10: 52, 58
Ch. 11: 3, 16, 28, 32, 33, 39
Section 11.4 will be included with Ch. 12.
We will introduce Ch. 11 by discussing
angular momentum for fixed axis rotation
first, since this makes a good transition
from Ch. 10.

Angular Momentum

For rotation about a fixed axis, angular momentum is easy to define as a rotational analogy:

$$p = mv \quad \text{becomes} \quad L = I\omega.$$

$$F = dp/dt \quad \text{becomes} \quad \tau = dL/dt$$

Momentum is conserved in the absence of external forces.

Angular momentum is conserved in the absence of external torques.

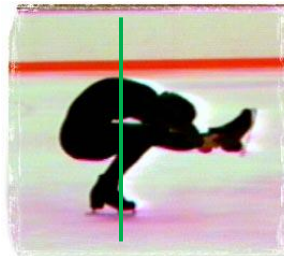
Figure Skating

- You can change the angular momentum of an object just by changing its shape.
- When a figure skater draws his body in, his moment of inertia decreases, increasing his angular velocity: $I_1 \omega_1 = I_2 \omega_2$.

Small I , fast ω



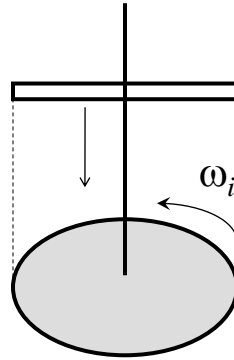
large I , slow ω



Angular Momentum Conservation

A stick of mass m is dropped as shown onto a rotating platter with mass M and angular speed ω_i . Find ω_f .

The stick is as long as the platter's diameter, and both are uniform.

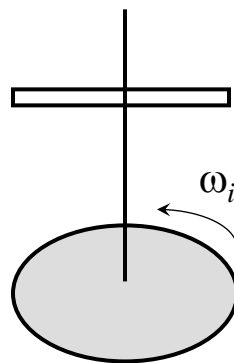


Rotational Inelastic Collision

Torques cancel about the central axis, since all forces are equal and opposite, and the axis is the same.

Initial angular momentum:

$$L = I_i \omega_i = \frac{1}{2} MR^2 \omega_i$$



Rotational Inelastic Collision

Final angular momentum:

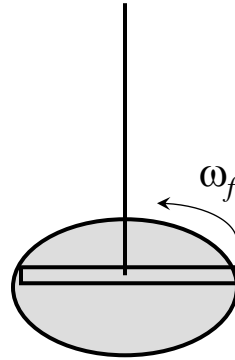
$$L = I_f \omega_f$$

$$I_f = (1/12) m(2R)^2 \text{ (stick)}$$

$$+ \frac{1}{2} MR^2 \text{ (disk)}$$

$$= [(1/3) m + \frac{1}{2} M] R^2.$$

$$\frac{1}{2} MR^2 \omega_i = [(1/3) m + \frac{1}{2} M] R^2 \omega_f$$



$$\omega_i = \left[\frac{2m}{3M} + 1 \right] \omega_f \Rightarrow \omega_f = \frac{3M\omega_i}{2m + 3M}$$

Rotational Inelastic Collision

This part was not used in class due to time constraints.

Energy is not conserved in an inelastic collision.

What fraction of the initial energy is lost in this one?

$$K_i = \frac{1}{2} I_i \omega_i^2$$

$$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (I_f \omega_f) \omega_f = \frac{1}{2} (I_i \omega_i) \omega_f$$

Divide: $\frac{K_f}{K_i} = \frac{\omega_f}{\omega_i} \left(= \frac{I_i}{I_f} \right)$

$$\frac{K_i - K_f}{K_i} = 1 - \frac{\omega_f}{\omega_i} = 1 - \frac{3M}{2m + 3M} = \frac{2m}{2m + 3M}$$