Physics 221

Department of Physics
The Citadel

Lecture Notes

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Rigid Body Motion, Part 3:
Torque, Rolling, Moving Axis

Announcements

- Homework Set 10: due Friday
  Problems: 4, 16, 21, 25, 30, 33, 34, 37 from sections 10.1 – 10.8

Today: Ch. 10.9 (rolling),

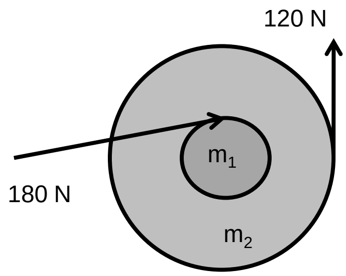
Next: Ch. 11: sec. 1 – 4 (angular momentum)

But we will discuss angular momentum for fixed axis rotation first, unlike the text, which does the general case first.

Example

A wheel of radius 24 cm and mass 5.0 kg is attached to an axle of radius 6.5 cm and mass 1.2 kg. Forces are applied as shown.

Find the net torque and angular acceleration.



Example

τ = 120N (0.24 m) – 180 N (0.065 m)

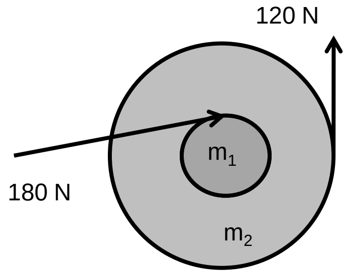
= 17.1 Nm

$I = \frac{1}{2} m_1 r_1^2 + \frac{1}{2} m_2 r_2^2$

= 0.1465 kg m^2.

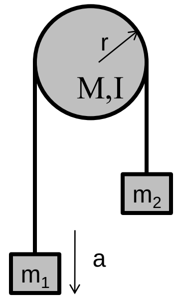
$\alpha = \tau / I$ = 117 rad/s^2.

Direction: ccw.



Inside: $r_1 = 0.065$ m
$m_1 = 1.2$ kg
Outside: $r_2 = 0.240$ m
$m_2 = 5.0$ kg

Atwood Machine



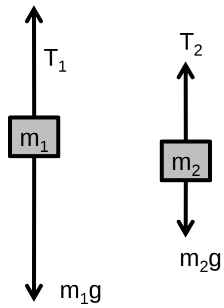
If the Atwood machine is built with a pulley having moment of inertia $I = cMr^2$, what is the acceleration of the masses?

Newton's Law for the blocks:

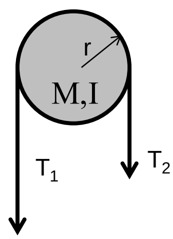
$$m_1 g - T_1 = m_1 a$$

$$T_2 - m_2 g = m_2 a.$$

We cannot assume $T_1 = T_2$.



Atwood Machine



Torque equation:

$\tau = I\alpha$.

Net torque: $\tau = (T_1 - T_2)r$

Angular acceleration: $a = r\alpha$

$(T_1 - T_2)r = Ia/r = cMr^2 a/r = cMra$

$$T_1 - T_2 = cMa$$

$$m_1 g - T_1 = m_1 a$$

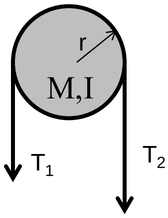
$$T_2 - m_2 g = m_2 a$$

$$m_1 g - m_2 g = (m_1 + m_2 + cM)\, a$$

$$a = \frac{m_1 - m_2}{m_1 + m_2 + cM} g$$

Atwood Machine

What are the two tensions?



$T_1 = m_1\,(a + g)$

$T_2 = m_2\,(g - a)$

$$a = \frac{m_2 - m_1}{m_1 + m_2 + cM}\,g$$

$$T_1 = \frac{m_1(2m_2 + cM)g}{m_1 + m_2 + cM} \qquad T_2 = \frac{m_2(2m_1 + cM)g}{m_1 + m_2 + cM}$$

$$T_2 - T_1 = \frac{cM(m_2 - m_1)g}{m_1 + m_2 + cM} = cMa$$

Rolling

When a wheel is rolling, it goes a distance $s = 2\pi R = vt$ every time t it rolls once.

In the same time, it turns through an angle

$2\pi = \omega t$. Dividing gives $R = v/\omega$, or $v = R\omega$.

The axle goes at speed v.

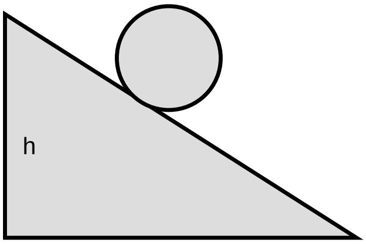
A point on top goes at speed $v + R\omega = 2v$.

A point on the ground goes at speed

$v - R\omega = 0$! That means the point on the ground is instantaneously at rest.

Rolling Down an Incline

If a marble, hoop, and cylinder roll down an inclined plane, which goes fastest?



The shape affects the moment of inertia. But do the mass or size matter?

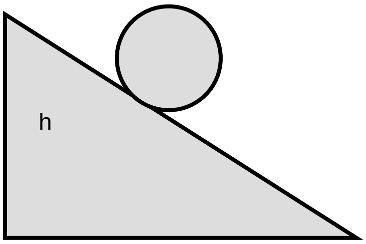
For each shape, $I = cMR^2$.

Marble: $c = 2/5$

Hoop: $c = 1$, Cylinder: $c = ½$.

Rolling Down an Incline

A wheel of moment of inertia $I = cMR^2$ (with c a constant) rolls down an incline starting from rest. What is the speed at the bottom?



$Mgh = ½\, Mv^2 + ½\, I\omega^2$
$= ½\, (M + I/R^2)v^2$.

$gh = ½\, (1+c)v^2$,

$v = (2gh/(1+c))^{1/2}$

The smaller c wins:
The marble.

M and R don't matter.

Rigid Body Motion

Force causes the CM of an object to accelerate:

$$\mathbf{F} = m\mathbf{a}.$$

Torque causes rotation of the object about the CM:

$$\tau = I\,\alpha.$$

For now we will consider only “fixed axis rotation”, where the direction of the axis doesn’t change. Later, we will generalize, partially. A complete treatment of general rotational motion is beyond the scope of this course.

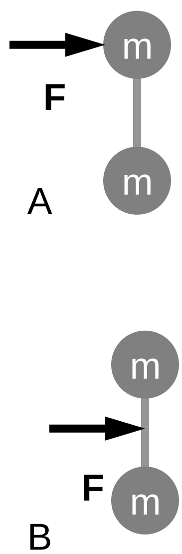
Dumbbell

A force F is applied for time t to a dumbbell in one of two ways shown.

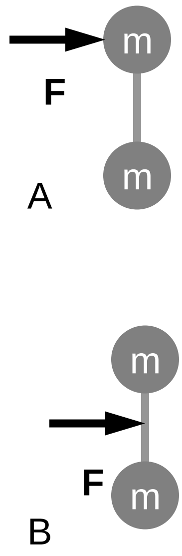
Which gives the greater acceleration to the center of mass?

(a) A (b) B

(c) the same

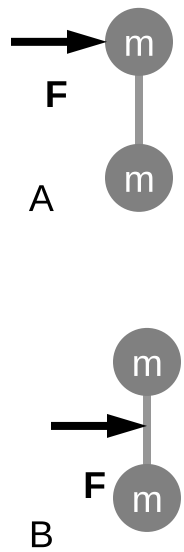


Dumbbell



- The acceleration is $a = F/2m$ in either case.
- The momentum acquired is $2mv = Ft$ and $v = F/2m.$
- It doesn't matter where the force is applied. Only the magnitude and direction affect the change in linear momentum.

Dumbbell



Which force does more work?

(a) A (b) B

(c) the same

A) $W = ½ (2m)v^2 + ½ I\omega^2$

$= mv^2 + ½ (2mR^2)(v/R)^2$

$= 2mv^2.$

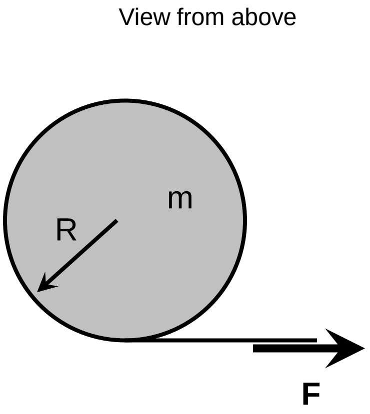
B) $W = mv^2.$

Pull the Spool

A solid uniform disk of mass m and radius r rests flat on a frictionless surface.

A string is wrapped around the disk, and does not slip when pulled.

How fast is the disk moving when it has been pulled a distance x?



Pull the Spool

Work-Energy theorem:

$\frac{1}{2} mv^2 = Fx$

$v^2 = 2Fx/m$

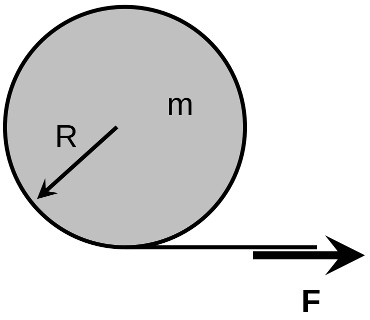
How fast is it spinning at that point?

$\tau = I\alpha$, with $\tau = RF$ and $I = \frac{1}{2} mR^2$.

$\omega = \alpha\, t = \alpha\, mv/F$.

$\alpha = RF/(\frac{1}{2}mR^2) = 2F/mR$.

$\omega = 2v/R$.



Pull the Spool

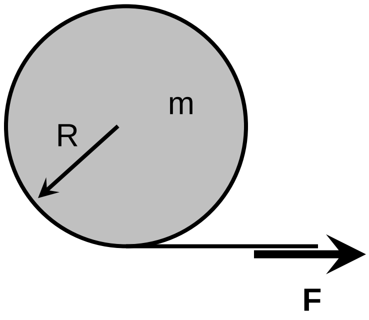
How much string has unwrapped from around the rim at that point?

Use the relation

$R\omega = 2v$.

The length of string is the distance the rim turns:

$L = R\theta = R\omega t = 2vt = 2x$.



Pull the Spool

This slide was not used in class.

The distance your **hand** moved while pulling the string is $L + x$.

The work done by your hand was $F(L + x) = 3Fx$.

Where, exactly, did it go?

The translational energy at x was

$\frac{1}{2} mv^2 = Fx$.

The rotational energy at x was

$\frac{1}{2} I\omega^2 = \frac{1}{2} (\frac{1}{2}mR^2) (2v/R)^2 = mv^2$ (twice as much)

Compare rotational work: $\tau\theta = RF\theta = 2Fx$.

Twice as much work went into spinning the spool as into moving it across the table.