Physics 221

Department of Physics
The Citadel

Lecture Notes

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October 19, 2008

Momentum and Impulse

Part 2

Announcements

- Homework Set 9: Up later today, due Friday (due to internet problems).

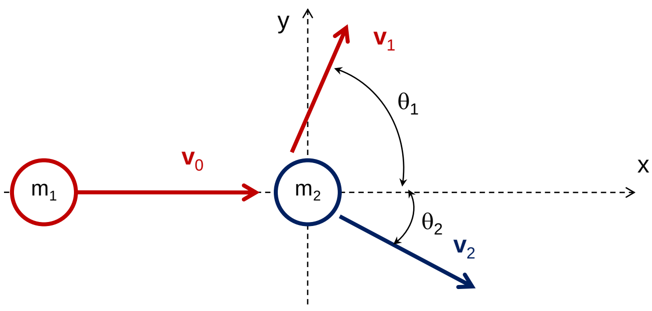
  #8, 16, 18, 20, 26, 29, 37, 43
- Exam 2 is being graded – back Wednesday (next problem discussion day)
- Current Chapter: 9 – Momentum and Collisions.

  Last Week: sec. 1 – 3 [4]

  Today: sec. 4 – 6 Skipping: sec. 7 – 8

2D Collision

Particle 1 of mass m_1 traveling at speed $v_0 = 66$ m/s strikes a stationary particle 2 mass $m_2 = 2m_1$. As a result, particle 1 is deflected through an angle $\theta_1 = 77^o$ and has final speed $v_1 = 28$ m/s. Find the velocity of particle 2 after the collision. Give its speed v_2 and angle θ_2.



2D Collision

Momentum conservation: $m_1 \mathbf{v}_0 = m_1 \mathbf{v}_1 + 2m_1 \mathbf{v}_2$

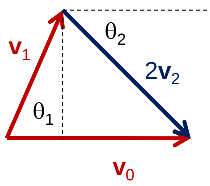
Then $\mathbf{v}_0 = \mathbf{v}_1 + 2\mathbf{v}_2$. $2\mathbf{v}_2 = \mathbf{v}_0 - \mathbf{v}_1$

x: $v_{2x} = ½ (v_0 - v_1 \cos\theta_1)$

y: $v_{2y} = - ½\, v_1 \sin\theta_1$

$v_0 = 66$ m/s, $v_1 = 28$ m/s , $\theta_1 = 77^o$

$v_{2x} = 29.9$ m/s, $v_{2y} = - 13.6$ m/s.

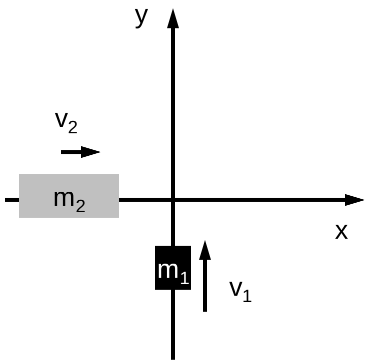


$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = 32.8 \text{ m/s}$$

$\tan\theta_2 = v_{2y} / v_{2x} = - 13.6/29.9 = - 0.455$, $\theta_2 = - 24.5^o$

2D Inelastic Collision

- A car of mass 2000 kg traveling north at 40 mph strikes a truck of mass 6000 kg traveling east at 20 mph. What speed and direction do they travel if they stick together?



2D Inelastic Collision

The initial x component of momentum is

$$P_x = m_2 v_2.$$

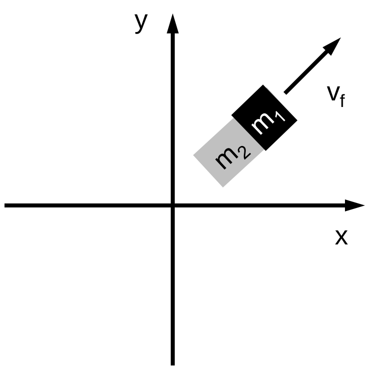
The initial y component of momentum is

$$P_y = m_1 v_1.$$

After the collision,

$$P_x = (m_1 + m_2)\, v_f \cos\theta = m_2 v_2$$

$$P_y = (m_1 + m_2)\, v_f \sin\theta = m_1 v_1$$



2D Inelastic Collision

$v_f \cos\theta = m_2 v_2 / (m_1 + m_2)$

$= 0.75\, v_2 = 15$ mph

$v_f \sin\theta = m_1 v_1 / (m_1 + m_2)$

$= 0.25\, v_1 = 10$ mph

$v_1 = 40$ mph
$v_2 = 20$ mph

$m_1 = 2000$ kg
$m_2 = 6000$ kg

$\tan\theta = 2/3, \quad \theta = 34^{\circ}$

and

$v_f = 10$ mph / $\sin 34^{\circ} = 18$ mph

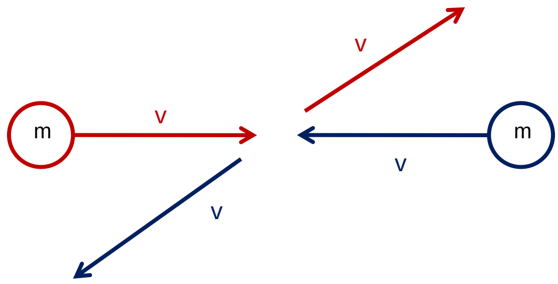
2D Elastic Collision

2D elastic collisions are best solved using momentum conservation and energy conservation. This gives three equations, which can determine three unknowns.

It is still true that the relative speed is the same before and after an elastic collision, but this is less helpful in 2D collisions since the direction of the relative velocity is usually unknown.

2D Elastic Collision

Example: Two identical masses collide with opposite velocities. If the collision is elastic, they still have opposite velocities after the collision, but the direction is undetermined.



2D Elastic Collision

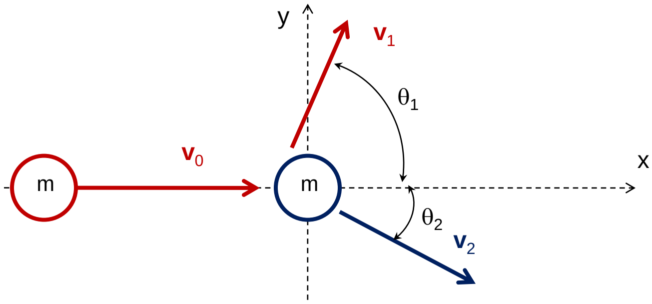
2D elastic collisions are usually best solved using momentum conservation and energy conservation.

This gives three equations (momentum has two components), which can determine three unknowns.

It is still true that the relative speed is the same before and after the collision, but this is less helpful in 2D collisions since the direction of the relative velocity after the collision is usually unknown.

2D Elastic Collision

A hockey puck traveling at speed $v_0 = 5.0$ m/s strikes an identical puck initially at rest and its speed is reduced to $v_1 = 3.0$ m/s. Find the speed of the v_2 of the other puck, and the angles of the two pucks, assuming the collision is elastic.



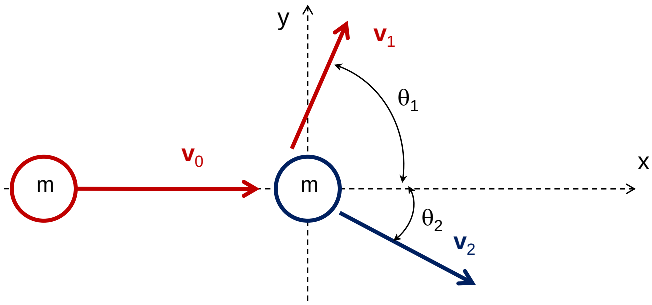
2D Elastic Collision

Momentum conservation: $m\mathbf{v}_0 = m\mathbf{v}_1 + m\mathbf{v}_2$

Energy conservation: $\frac{1}{2}\, mv_0^2 = \frac{1}{2}\, mv_1^2 + \frac{1}{2}\, mv_2^2$.

We can cancel the masses and relate velocities:

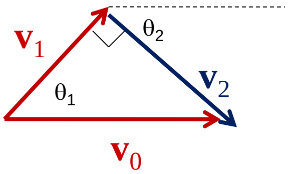
$$\mathbf{v}_0 = \mathbf{v}_1 + \mathbf{v}_2 \qquad v_0^2 = v_1^2 + v_2^2$$



2D Elastic Collision

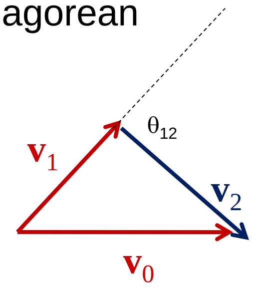
Known: $\mathbf{v}_0 = \mathbf{v}_1 + \boldsymbol{v}_2$ $v_0^2 = v_1^2 + v_2^2$

The first equation says that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_0$ are sides of a triangle, and the second says that the sides obey Pythagoras' Theorem with $v_0 = 5$ m/s as the hypotenuse of a right triangle with sides $v_1 = 3$ m/s and v_2. Thus, the geometry must be as shown:



Proof

Proof that a triangle obeying the Pythagorean formula must be a right triangle:



Given: $\mathbf{v}_0 = \mathbf{v}_1 + \boldsymbol{v}_2$, $v_0^2 = v_1^2 + v_2^2$

Use the dot product:

$$v_0^2 = \mathbf{v}_0 \cdot \mathbf{v}_0 = (\mathbf{v}_1 + \boldsymbol{v}_2) \cdot (\mathbf{v}_1 + \boldsymbol{v}_2)$$

$$= v_1^2 + v_2^2 + 2\, \mathbf{v}_1 \cdot \mathbf{v}_2$$

This is consistent with the Pythagorean formula only $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$. Since $\mathbf{v}_1 \cdot \mathbf{v}_2 = v_1\, v_2 \cos\theta_{12}$, this implies that $\cos\theta_{12} = 0$, so $\theta_{12} = 90^{\circ}$.

2D Elastic Collision

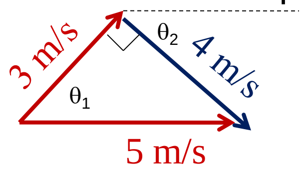
In fact, $v_0^2 = v_1^2 + v_2^2$ immediately gives v_2:

$$v_2 = \sqrt{v_0^2 - v_1^2} = 4\text{m/s}$$

The angles are then found by trigonometry:

$\theta_1 = \tan^{-1}(3/4) = 36.9^o$, $\theta_2 = 90^o - \theta_1 = 53.1^o$.

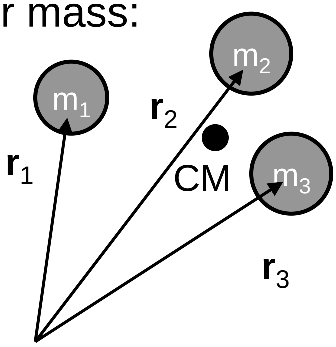
The fact that the final velocities $\mathbf{v}_1$, $\mathbf{v}_2$ are perpendicular is a general property of equal mass collisions in 2d with one particle initially at rest.



Center of Mass

The center of mass of a collection of particles is the vector sum of their positions weighted by their mass:

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_1 + m_2 + m_3}$$



In general notation:

$$\mathbf{r}_{CM} = \frac{\sum_i m_i\mathbf{r}_i}{\sum_i m_i}$$

Center of Mass in Collisions

Taking the derivative of the CM position gives the CM velocity

$$\mathbf{v}_{\mathrm{CM}} = \frac{d\mathbf{r}_{\mathrm{CM}}}{dt} = \frac{\frac{d}{dt}\sum_i m_i \mathbf{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \mathbf{v}_i}{\sum_i m_i} = \frac{\mathbf{P}_{\mathrm{total}}}{M_{\mathrm{total}}}$$

Since $\mathbf{P}_{\mathrm{total}}$ is constant in the absence of external forces, so is $\mathbf{v}_{\mathrm{cm}}$.

Take another derivative and multiply by the mass:

$$\mathrm{M}_{\mathrm{total}}\ \mathbf{a}_{\mathrm{cm}} = d\mathbf{P}/dt = \mathbf{F}_{\mathrm{ext}}.$$

This gives Newton's Law for a system of particles, or a general extended object.

Motion of Extended Objects

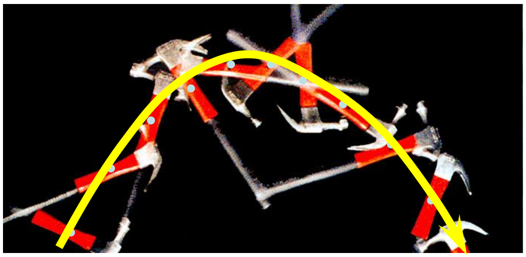
The motion of extended objects or collections of particles is such that the CM obeys Newton's 2nd Law.

Motion of the Center of Mass

The CM of a wrench sliding on a frictionless table will move in a straight line because there is no external force. In this sense, the wrench may be though of as a particle located at the CM.

Motion of the Center of Mass

If a hammer is thrown, its CM follows a parabolic trajectory under the influence of gravity, as a point object would.



Example – Exploding Projectile

If a projectile explodes into several pieces, and you follow the pieces along their trajectories, the CM of the pieces will follow the same path the projectile would have followed (assuming it is close enough to earth that the net gravitational force is still Mg).