Physics 221

Department of Physics
The Citadel

Lecture Notes

S. Yost
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Momentum and Impulse

Part 1

Announcements

- Homework Set 9: Up later today, due next Wednesday. #8, 16, 18, 20, 26, 29, 37, 43
- Exam 2 is being graded – back next Wednesday (next problem discussion day)
- Current Chapter: 9 – Momentum and Collisions.

  Today: sec. 1 – 3 [4]

  Monday: sec. 4 – 6

  Skipping: sec. 7 – 8

Force Acting Over Time

When a constant force acts on a mass over a length of time, the velocity changes.

Starting from rest,

$$F = m\,a$$
$$= m\,(v/t) = (mv)/t.$$

Then

$$mv = F\,t.$$

We call the left-hand side the **momentum**, and the right-hand side the **impulse**.

Momentum and Impulse

- The momentum is often written as $p = mv$.
- The units of momentum are kg m/s = N s.
  - Sadly, momentum does not have a better name, in spite of being one of the most fundamental quantities in all of physics!
- Impulse is written $J = Ft$ (for constant force).
- In general, $\Delta p = \int F dt = F_{\text{avg}}\, t = J$.

More than One Dimension

In general, the velocity has a direction, so momentum is a vector: $\mathbf{p} = m\,\mathbf{v}$

So is the impulse: $\Delta\,\mathbf{p} = \int \mathbf{F}\,dt$

Newton's Law can be written in terms of momentum:

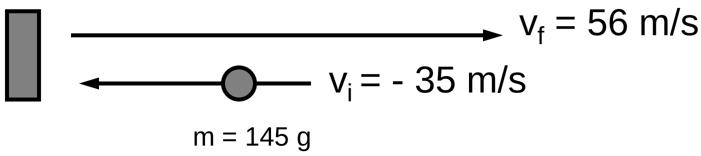
$$\mathbf{F} = d\mathbf{p}/dt.$$

This is the original form of Newton's law. He called momentum the "quantity of motion".

Example

- A 145-gram baseball is pitched at 35 m/s and hit on a line drive straight back at the pitcher at 56 m/s. If the bat was in contact with the ball for 5.0 ms, what was the average force of the bat on the ball?

Example



- It is important to keep track of the direction of the velocity: I wrote the velocity to the left as negative.

Example

The change in momentum is

$$\Delta p = m\,(v_f - v_i)$$
$$= 0.145 \text{ kg } (56 \text{ m/s} - (-35 \text{ m/s}))$$
$$= 13.2 \text{ kg m/s.}$$

The momentum change Δp = 13.2 kg m/s is the impulse on the ball.

It occurred in time t = 0.0050 s, so the average force was

$$F = \Delta p / t = 2.64 \text{ kN.}$$

Momentum Conservation

Newton's laws were obtained in part by studying collisions between masses.

It was found that when objects collide, with no external forces, momentum is conserved.

In fact, as long as you take a big enough system that all relevant forces are included, momentum is conserved. In this sense, momentum conservation is one of the most fundamental properties of the universe.

All known fundamental interactions conserve momentum. This can be connected to the fact that space is the same everywhere, with the same laws of physics.

Collisions

The impulses on two colliding objects are equal and opposite:

$$\Delta \mathbf{p}_1 = -\Delta \mathbf{p}_2, \qquad \mathbf{F}_{21}t = -\mathbf{F}_{12}\,t.$$

We recognize this as Newton's 3rd law. This is where it came from.

Or, turning it around, the conservation of momentum for an isolated system is a consequence of Newton's 3rd law: since all internal forces cancel exactly, so do all internal momentum changes.

Car and Truck

A small car has a head-on collision with a large truck and they stick together.

Which vehicle has the bigger change in momentum?

- Neither – their change in momenta are equal and opposite.

Which vehicle has the greater acceleration on impact?

- The force is the same (3rd law), so the car has more acceleration.

Example

For example, a railroad car of mass $M_1 = 1600$ kg could travel at $v_i = 2.4$ m/s and strike a second railroad car of mass $M_2 = 3200$ kg.

If the cars hitch together, how fast do they travel together after hitching?

Example

Before joining, the first car was the only one moving, so the total momentum was

$$p_i = M_1 v_i$$

After joining, the total momentum is

$$p_f = (M_1 + M_2) v_f$$

Momentum conservation: $M_1 v_i = (M_1 + M_2) v_f$

Then the final velocity of the hitched cars is

$$v_f = \frac{M_1 v_i}{M_1 + M_2} = v_i/3 = 0.80 \text{ m/s}$$

Example

- How much energy was transformed into microscopic vibrational motion in the collision?

Initial KE = $\frac{1}{2} M_1 v_i^2$

$= \frac{1}{2}$ (1600 kg)(2.4 m/s)^2 = 4608 J.

Final KE = $\frac{1}{2} (M_1+M_2)v_f^2$

$= \frac{1}{2}$ (4800 kg)(0.80 m/s)^2 = 1536 J.

Energy transformed = 4608 J – 1536 J

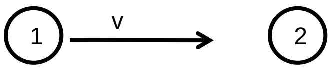
= 3072 J

Elasticity of Collisions

- A collision where things stick together (zero relative velocity after colliding) is called **inelastic**.
- One where two objects collide and have the same relative speed before and after the collision is called **elastic**.
- Many other possibilities also exist, of course. The speed of separation can be less or greater than the speed of approach.

Elastic Collision

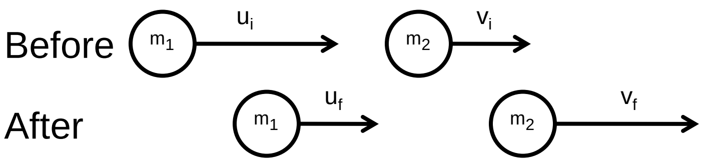
- A simple example is when one particle of mass m and velocity v hits an identical particle initially at rest.



- The first mass stops and the second moves away at velocity v. Energy is conserved, and the relative speed of the particles is the same before and after the collision.

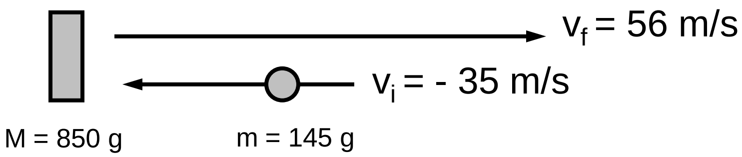
Elastic Collision

- In general, m_1 and m_2 need not be the same, but the relative speed is the same before and after the collision.



- In an elastic collision, $u_i - v_i = v_f - u_i$. Note the reversal in the order: the particles are approaching before the collision, but separating afterward.

Bat and Ball Example



Consider the example where a bat was swung at a ball pitched at 35 m/s, hitting the ball back on a line-drive at 56 m/s.

What was the speed of the bat before and after the collision if the bat has mass 850 g and the collision is elastic?

Treat the bat and ball as an isolated system, with no external force applied by the batter.

Bat and Ball Example

Initial and final velocities of the bat: u_i, u_f

Mass of the bat: M, mass of the ball: m

Momentum conservation: $Mu_i + mv_i = Mu_f + mv_f$.

Elastic collision: $u_i - v_i = v_f - u_f$.

(Does the sign make sense?)

Put the bat variables on the left:

$$M(u_i - u_f) = m(v_f - v_i) = \Delta p = 13.2 \text{ kg m/s.}$$

(calculated in the earlier example)

$$u_i + u_f = v_f + v_i = (56 - 35) \text{ m/s} = 21 \text{ m/s.}$$

Bat and Ball Example

Divide the momentum conservation equation by M:

$$u_i - u_f = (13.2 \text{ kg m/s})/(0.850 \text{ kg}) = 15.5 \text{ m/s}$$

$$u_i + u_f = 21 \text{ m/s}$$

Add: $2u_i = (15.5 + 21) \text{ m/s} = 36.5 \text{ m/s}$

Initial velocity of bat: $u_i = 18.3$ m/s.

Final velocity of bat: $u_f = 21 \text{ m/s} - u_i = 2.7$ m/s.

Kinetic Energy In Collisions

How does the kinetic energy change in an elastic collision?

Consider the bat and ball. We had two equations:

Momentum conservation: $M(u_i - u_f) = m(v_f - v_i)$.

Elastic collision: $u_i + u_f = v_f + v_i$.

Multiply the left and right sides of these equations:

$$M(u_i^2 - u_f^2) = m(v_f^2 - v_i^2).$$

Divide by 2 and rearrange:

$$\tfrac{1}{2} Mu_i^2 + \tfrac{1}{2} mv_i^2 = \tfrac{1}{2} Mu_f^2 + \tfrac{1}{2} mv_f^2.$$

Elastic collisions conserve energy.

Definition of Elastic Collision

An elastic collision conserves energy and momentum.

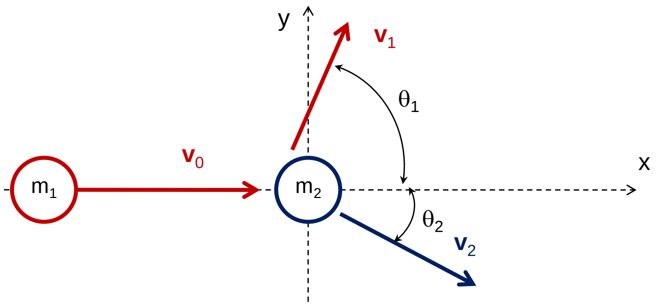
I originally **defined** elastic collisions as ones in which the relative speed of the two objects doesn't change in the collision.

This is close to Newton's original definition, but the book takes the more common approach as defining an elastic collision as one that conserves energy.

For more than two objects colliding, it doesn't make much sense to talk about the relative speed any more, so the definition in terms of energy is more general.

2D Collision

Particle 1 of mass m_1 traveling at speed v_0 = 66 m/s strikes a stationary particle 2 mass $m_2 = 2m_1$. As a result, particle 1 is deflected through an angle $\theta_1 = 77^o$ and has final speed v_1 = 28 m/s. Find the velocity of particle 2 after the collision. Give its speed v_2 and angle θ_2.



2D Collision

Momentum conservation: $m_1 \mathbf{v}_0 = m_1 \mathbf{v}_1 + 2m_1 \mathbf{v}_2$

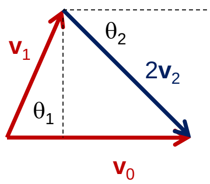
Then $\mathbf{v}_0 = \mathbf{v}_1 + 2\mathbf{v}_2$. $2\mathbf{v}_2 = \mathbf{v}_0 - \mathbf{v}_1$

x: $v_{2x} = \frac{1}{2}(v_0 - v_1 \cos\theta_1)$

y: $v_{2y} = -\frac{1}{2} v_1 \sin\theta_1$

v_0 =66 m/s, v_1 = 28 m/s , $\theta_1 = 77^o$

v_{2x} = 29.9 m/s, v_{2y} = 13.6 m/s.



$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = 32.8 \text{ m/s}$$

$\tan\theta_2 = v_{2y} / v_{2x} = -13.6/29.9 = -0.455$, $\theta_2 = -24.5^o$