Physics 221

Sections 1 and 2

S. Yost

October 9

NonConservative Forces, Power

Announcements

- Exam 2 will be held next Wednesday, covering Chapters 5 – 8. Only sections 1 and 2 of Chapter 6 are included.
- Today, we will discuss topics from sec. 8.3 – 8.5.
- The next problem set is due Monday:
- Ch. 7: 43, 45;
  Ch. 8: 3, 5, 7, 35, 41, 54.

Non-Conservative Forces

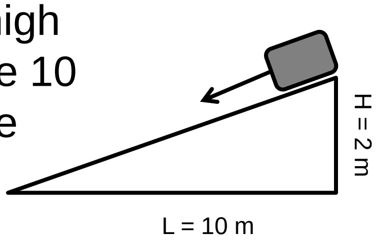
- Friction is not a conservative force. Any work done by friction or other non-conservative forces changes the total energy of a system.

$$\Delta E = \Delta K + \Delta U = W_{\text{nc}}$$

Here, U is the potential energy due to all conservative forces acting on the system.

Sliding Object

A ramp with coefficient of friction μ = 0.10 is placed at the edge of a 2 m high platform, with the base 10 m from the edge of the platform.



A box slides down the platform, starting from rest. How fast is it moving at the bottom?

Sliding Object

Initial energy

$E_0 = U = mgH$

Final energy:

$E_f = K = ½\, mv^2$

$E_f = E_0 + W_{nc}$

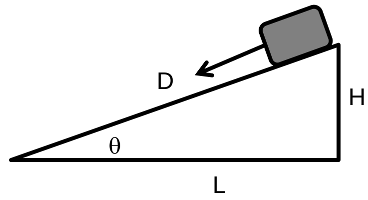
$W_{nc} = -D\, F_f$

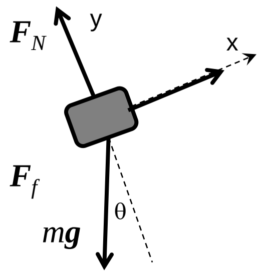
D = distance down ramp

$F_f = \mu F_N$

$F_N = mg \cos\theta$

$W_{nc} = -\mu Dmg \cos\theta = -\mu mgL$



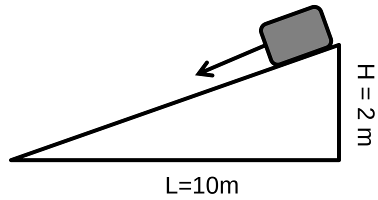


Sliding Object

$E_f = E_0 + W_{nc}$

$E_0 = mgH, \quad E_f = ½\, mv^2$

$W_{nc} = -\mu mgL$



$½\, mv^2 = mgH - \mu mgL$

$v^2 = 2g(H - \mu L)$

$= 2(9.8\ \text{m/s}^2)(2\text{m} - 0.1\text{x}10\text{m})$

$= 19.6\ \text{m}^2/\text{s}^2$

$v = 4.4$ m/s.

Friction: Example 2

- A block slides down the ramp shown, and slides a distance of 16 m along the floor before stopping. What is the coefficient of friction, assuming the ramp and floor are made of the same material?

Friction: Example 2

Initial energy: $E_0 = mgh$.

Final energy: $E_f = 0$.

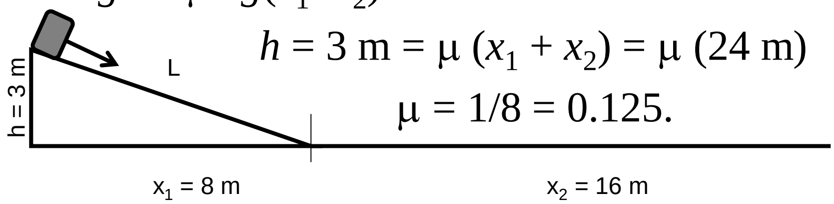
$\Delta E = -mgh = W_{nc}$.

Using the previous result, the work done by friction is $W_{nc} = -\mu mg(x_1+x_2)$.

$$mgh = \mu mg(x_1+x_2)$$

$$h = 3\text{ m} = \mu\,(x_1 + x_2) = \mu\,(24\text{ m})$$

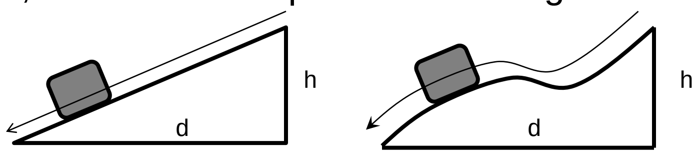
$$\mu = 1/8 = 0.125.$$



Path-Dependence

The next two slides are a conceptual note added after class.

The work done by gravity in the previous problems was $mg\Delta y$, which would be independent of the path, even if the ramp were not straight.



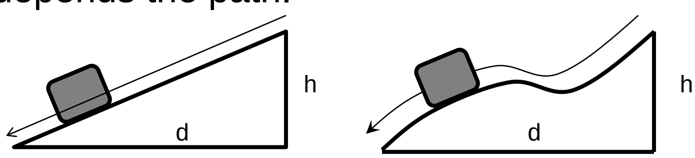
Without friction, $\Delta K = W_g = mgh$ in both cases, so the speed at the bottom would be the same.

In fact, the length d is irrelevant without friction.

Only the height h matters.

Path-Dependence

The work done by friction in the previous problems was $-\mu mg\Delta x$, but friction is **not** conservative, so this depends the path.

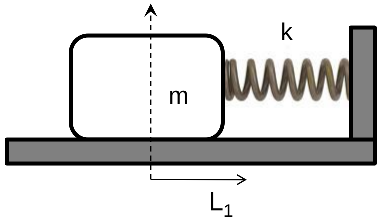


$W_{nc} = -\mu mgd$ $\quad\quad$ $W_{nc} = -????$

The frictional force is harder to calculate in the second case because $F_{\mathrm{N}} \neq mg \cos\theta$: the forces perpendicular to the track **do not** balance if it is not straight. (There is radial acceleration.)

Spring With Friction

A mass m is attached to a spring with spring constant k and compressed a distance L_1 If it is sliding on a table with coefficient of friction μ, how far beyond the equilibrium point does the spring extend when the mass bounces back?

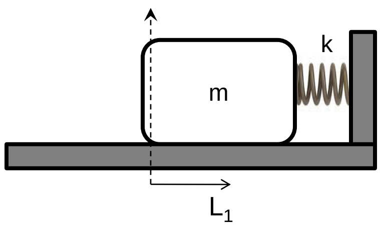


Given:
L_1, k, m, μ

Spring With Friction

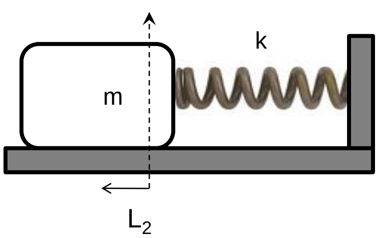
Compare the completely compressed spring to the completely extended spring. Both are at rest:

$K = 0$.



Initial energy:

$E_1 = ½ kL_1^2$



Final energy:

$E_2 = ½ kL_2^2$.

$E_2 – E_1 = W_{nc}$

Spring With Friction

$\frac{1}{2} k L_2^2 - \frac{1}{2} k L_1^2 = W_{nc}$

$W_{nc} = -\mu mg (L_1 + L_2).$

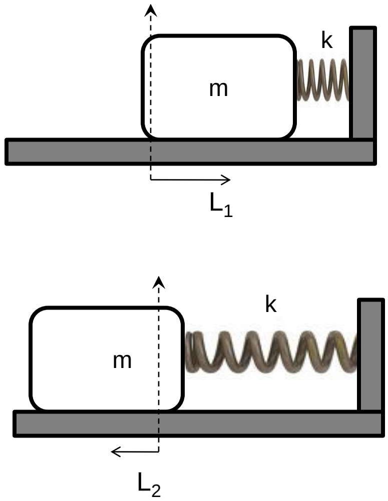
$\frac{1}{2} k (L_1^2 - L_2^2) = \mu mg (L_1 + L_2)$

$= \frac{1}{2} k (L_1 - L_2)(L_1 + L_2)$

$\frac{1}{2} k (L_1 - L_2) = \mu mg$

$L_1 - L_2 = 2\mu mg/k$

$\boxed{L_2 = L_1 - 2\mu mg/k.}$

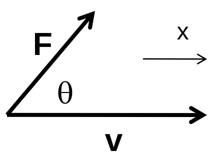


Power

Power is the rate of doing work: $P = \boxed{dW/dt.}$

$W = \int F_x dx, \quad dW/dx = F_x,$

$dx/dt = v.$



Chain rule: $F_x v = \frac{dW}{dx}\frac{dx}{dt} = \frac{dW}{dt} = P$

$P = F_x v = Fv \cos\theta = \boxed{\mathbf{F} \cdot \mathbf{v}}$

Power is measured in **Watts**: 1 W = 1 J/s.

Horsepower is another common unit: 1 hp = 746 W.

Accelerating Car

Suppose a car accelerates from zero to 15 m/s in 5.0 s. How long will it take to accelerate from 15 m/s to 30 m/s if the power output remains the same?

$Pt_1 = \Delta K_1 = ½\ m\ (15)^2 = 112.5\ m$ (SI units)

$Pt_2 = \Delta K_2 = ½\ m\ \{(30)^2 - (15)^2\} = 337.5\ m$

Divide the second equation by the first.

P and m cancel: $t_2/t_1 = 337.5/112.5$.

$t_2 = 30$ s.

Calculus Example

This example is closely related to problem 7 in the homework.

Suppose $F = ct^2$ acts on a mass m starting from rest. What is the kinetic energy after time t?

$$K = ½\ mv^2$$

$$v = \int_0^t a\,dt = \int_0^t \frac{F}{m}\,dt = \int_0^t \frac{ct^2}{m}\,dt = \frac{ct^3}{3m}$$

$$K = \tfrac{1}{2} mv^2 = \frac{m}{2}\left(\frac{ct^3}{3m}\right)^2 = \frac{c^2t^6}{18m}$$

Calculus Example

What is the power produced as a function of time?

$$P = Fv = ct^2\left(\frac{ct^3}{3m}\right) = \frac{c^2t^5}{3m}$$

Another way:

$$P = \frac{dW}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{c^2t^6}{18m}\right) = \frac{6c^2t^5}{18m} = \frac{c^2t^5}{3m}$$

Same result!