Physics 221

Sections 1 and 2

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October 7

Potential Energy and Energy Conservation

Announcements

- Exam 2 will be held next Wednesday, covering Chapters 5 – 8. Only sections 1 and 2 of Chapter 6 are included.
- Today we will discuss the last 2 sections of Ch. 7 and the first 3 sections of Ch. 8.
- Next time, we will finish Ch. 8
- The next problem set is due Monday:
- Ch. 7: 43, 45;

  Ch. 8: 3, 5, 7, 35, 41, 54.

Conservative Forces

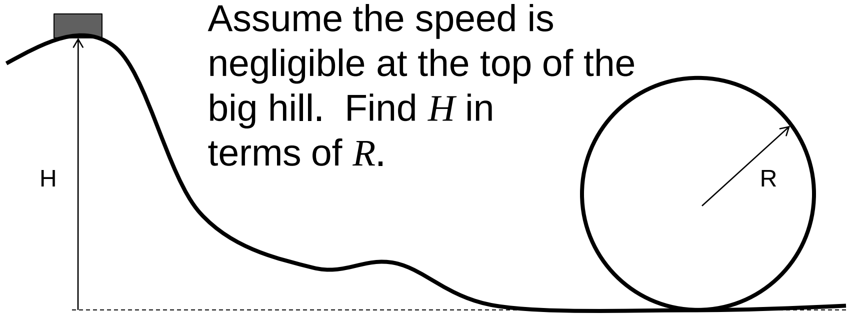
- Conservative forces have a potential energy function that keeps an account of the work done *against* the force.
- Gravity: The potential energy at height y is $U(y) = mgy$
- If only conservative forces act on an object, then the total energy is conserved:

$$E = K + U = \text{constant.}$$

Roller Coaster

A roller coaster follows the track shown. How high must the hill be for passengers to make it around the loop without falling against their seat belts?

Assume the speed is negligible at the top of the big hill. Find H in terms of R.

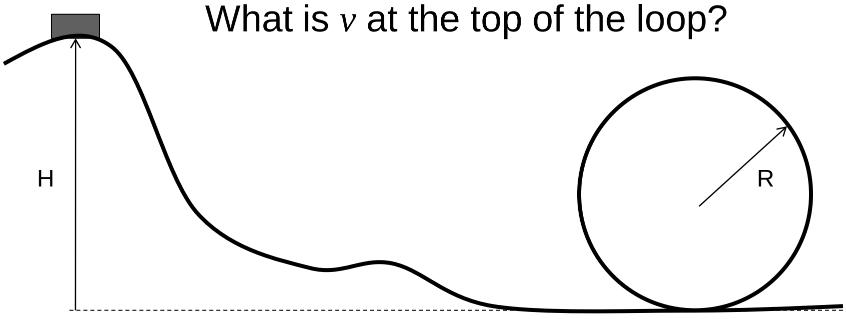


Roller Coaster

Energy at the top of the hill: $E = mgH$.

Energy at the top of the loop: $E = \frac{1}{2} mv^2 + 2mgR$

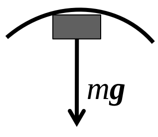
What is v at the top of the loop?



Roller Coaster

If the passengers are just about to lose contact with their seats, there is no normal force.

The net force is gravitational, and the acceleration is centripetal, so



$$F = mg = ma = mv^2/R.$$

$$v^2 = Rg.$$

Then at the top of the loop,

$$E = \tfrac{1}{2} mv^2 + 2mgR = \tfrac{1}{2} mgR + 2mgR$$

$$= 2.5\ mgR.$$

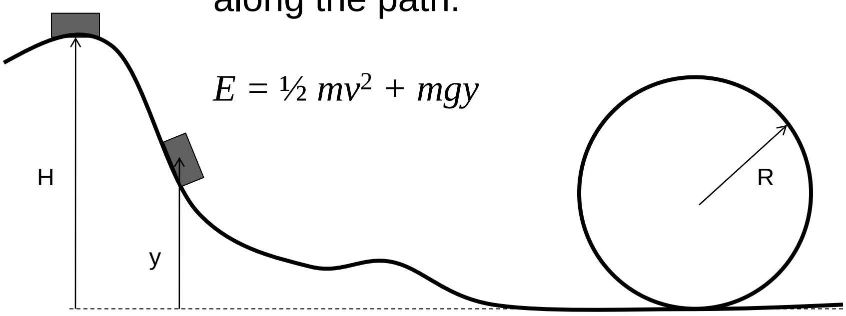
Roller Coaster

$E = mgH = 2.5\ mgR$

$H = 2.5R.$

Energy is conserved at every point along the path:

$E = \tfrac{1}{2}\, mv^2 + mgy$



Potential Energy and Force

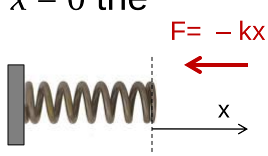
Potential energy is defined mathematically in terms of an integral:

$$U(x) = -\int_{x_0}^{x} F dx$$

The reference point x_0 may be chosen for convenience.

Example for the spring: $F = -kx$ with $x = 0$ the equilibrium position.

$$U(x) = -\int_{0}^{x} (-kxdx) = \tfrac{1}{2} kx^2$$



Potential Energy and Force

If you know the potential energy, you can find the force. Taking a derivative is the inverse of doing an integral:

$$\frac{dU}{dx} = -\frac{d}{dx}\int_{x_0}^{x} F dx = -F(x).$$

The second equality is the fundamental theorem of calculus.

This means that

$$F = -\frac{dU}{dx}$$

Potential Energy and Force

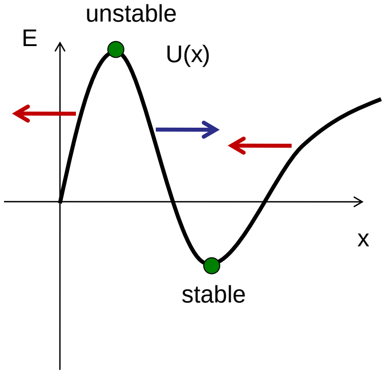
Interpretation of $F = -\frac{dU}{dx}$

If $U(x)$ slopes upward, the force is to the left.

If $U(x)$ slopes downward, the force is to the right.

If $U(x)$ is a local maximum or minimum, the force is zero. This is called an **equilibrium** position.

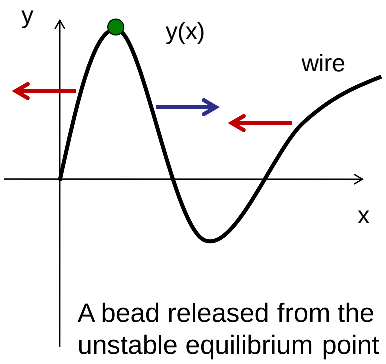
An equilibrium position is stable if it is a minimum, unstable if it is a maximum of $U(x)$.



Bead on a Wire

You can get a very literal picture of this by letting a bead slide on a frictionless wire, acted on by gravity, then $U = mgy$, so the energy diagram has the same shape as the wire.

A bead released from rest at the top would slide down toward either side, gaining kinetic energy as it fell. It would continue sliding as long as it stayed below its initial height.



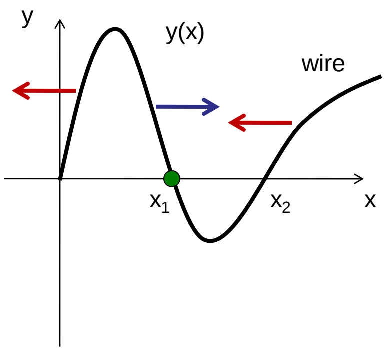
A bead released from the unstable equilibrium point would slide off the end of the wire.

Bead on a Wire

A bead released from the point x_1 point where $U = 0$ would feel a force to the right, causing it to accelerate to the right. The total energy would stay constant, $E = 0$, while U becomes negative.

Kinetic energy makes up the difference: $K = E - U(x)$.

The bead gains kinetic energy until it gets to the lowest point.



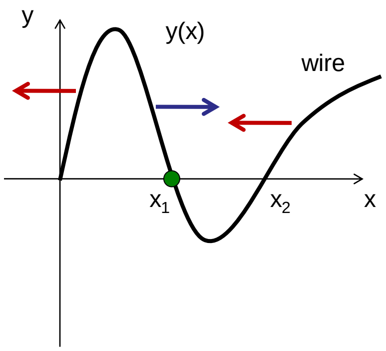
Bead on a Wire

The bead can't stop at the bottom: Its energy is conserved. Stopping here would leave it with E < 0.

The bead continues to x_2, where it must stop, because $U(x_2) = E = 0$, requiring $K = 0$.

But it can't stay there: the force is now to the left, forcing the bead back.

The bead continues to oscillate back and forth indefinitely, in the absence of friction.



Spring

A spring works similarly. If you pull a spring out to length L and release it, it has stored energy $E = ½\, kL^2$, which is conserved.

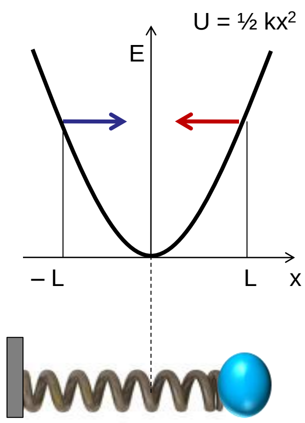
At any extension x, an attached mass m has energy.

$$E = ½\, kL^2 = ½\, kx^2 + ½\, mv^2.$$

$$mv^2 = k\,(L^2 - x^2)$$

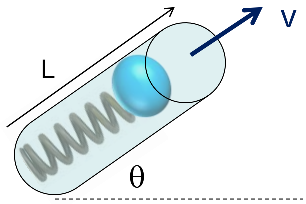
The speed of the mass at extension x is

$$v = \sqrt{\frac{k}{m}\left(L^2 - x^2\right)}$$



Spring Loaded Cannon

A miniature "cannon" can be loaded by compressing a spring the length of the barrel as the ball is loaded.   If the spring constant is 80 N/m, the barrel is 25 cm long, and the ball has a mass of 400 g, what is the launch speed as a function of the firing angle θ?



Spring Loaded Cannon

Energy when spring is compressed:

$E = \frac{1}{2} kL^2$.

Energy when ball launches:

$E = mgh + \frac{1}{2} mv^2 = mgL \sin\theta + \frac{1}{2} mv^2$.

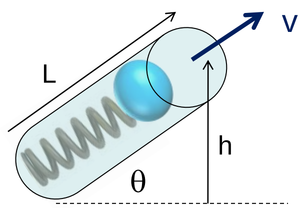
Energy conservation:

$\frac{1}{2} mv^2 = \frac{1}{2} kL^2 - mgL \sin\theta$

$v^2 = (k/m)L^2 - 2gL \sin\theta$

$= 12.5 - 4.9 \sin\theta$ in m^2/s^2.

$k = 80$ N/m
$m = 0.4$ kg
$L = 0.25$ m



$$v = \sqrt{12.5 - 4.9\sin\theta} \quad \text{m/s}$$

Car and Spring

Suppose we want to bring a 1000 kg vehicle traveling 30 m/s to rest using a spring, and want the maximum deceleration of the passengers to be $5g$'s. ($g = 9.8$ m/s^2)

What should be the spring constant and minimum uncompressed length of the spring?

Car and Spring

Energy conservation: $\frac{1}{2} mv^2 = \frac{1}{2} kx^2$.

How to use the acceleration information?

$$F = -kx = ma, \quad a = -kx/m.$$

Maximum deceleration at maximum compression.

For given $a = -5g$, $x = 5mg/k$.

Substitute this for x in the original equation:

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2 = \frac{1}{2} (5mg)^2/k.$$

$$k = m(5g/v)^2 = 2668 \text{ N/m}.$$

Compression:

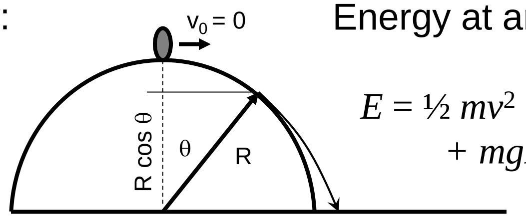
$$x = 5mg/k = 18.4 \text{ m}. \quad (\textit{very} \text{ long!})$$

Skating Off Igloo

An Eskimo skates off an igloo of radius R, starting at from rest at the top. At what angle θ does he lose contact with the igloo? Neglect friction.

Initial energy:

$E = mgR$



Energy at angle θ:

$E = \frac{1}{2} mv^2 + mgR \cos\theta$

Skating Off Igloo

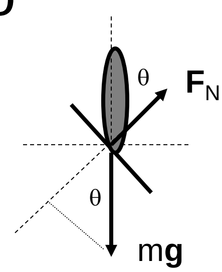
example used in 8AM class only

Total force in radial direction:

$mg \cos\theta - F_N = ma = mv^2/R$

At the point of departure, $F_N = 0$.

$v^2 = Rg \cos\theta$.



Free body diagram

Energy conservation:

$\frac{1}{2} mv^2 + mgR \cos\theta = mgR.$

$1.5gR \cos\theta = gR$

$\cos\theta = 2/3$

$\theta = 48.2^o$.