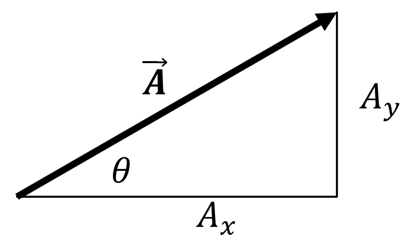
Equations for Physics 221

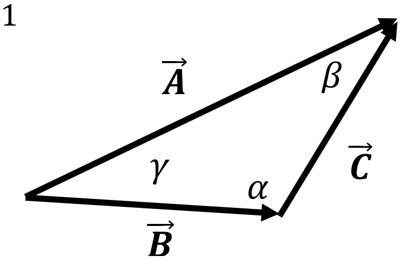
Mathematics

$$Ax^2 + Bx + C = 0 \quad \Rightarrow \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$



$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}$ $\quad A_x = A\cos\theta$ $\quad A_y = A\sin\theta$

$A = \sqrt{A_x^2 + A_y^2}$ $\quad \tan\theta = \frac{A_y}{A_x}$ $\quad \sin^2\theta + \cos^2\theta = 1$



$\vec{\boldsymbol{A}} = \vec{\boldsymbol{B}} + \vec{\boldsymbol{C}}$ $\quad A_x = B_x + C_x$ $\quad A_y = B_y + C_y$

$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = AB\cos\gamma = A_xB_x + A_yB_y$

$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = AB\sin\gamma\,\hat{\boldsymbol{k}} = (A_xB_y - A_yB_x)\hat{\boldsymbol{k}}$

$C^2 = \left(\vec{\boldsymbol{A}} - \vec{\boldsymbol{B}}\right)^2 = A^2 + B^2 - 2AB\cos\gamma$ $\quad \frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$

Circle: circumference $= 2\pi r$, area $= \pi r^2$

Sphere: surface area $= 4\pi r^2$, volume $= \frac{4}{3}\pi r^3$

$\frac{dx^a}{dx} = ax^{a-1}$, $\quad \frac{d}{dx}\sin ax = a\cos ax$, $\quad \frac{d}{dx}\cos ax = -a\sin ax$

$\int x^a dx = \frac{x^{a+1}}{a+1}$, $\quad \int \sin ax\,dx = -\frac{1}{a}\cos ax$, $\quad \int \cos ax\,dx = \frac{1}{a}\sin ax$

Kinematics

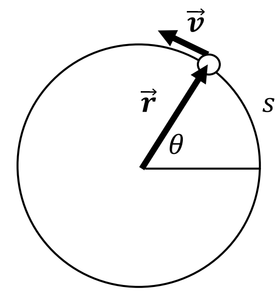
$\vec{\boldsymbol{v}}_{\text{avg}} = \frac{\Delta\vec{\boldsymbol{r}}}{\Delta t}$ $\quad \vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{r}}}{dt}$ $\quad \vec{\boldsymbol{a}}_{\text{avg}} = \frac{\Delta\vec{\boldsymbol{v}}}{\Delta t}$ $\quad \vec{\boldsymbol{a}} = \frac{d\vec{\boldsymbol{v}}}{dt}$

Constant acceleration: $\vec{\boldsymbol{v}}_f = \vec{\boldsymbol{v}}_i + \vec{\boldsymbol{a}}t$, $\quad \vec{\boldsymbol{r}}_f = \vec{\boldsymbol{r}}_i + \vec{\boldsymbol{v}}_i t + \frac{1}{2}\vec{\boldsymbol{a}}t^2$,

$\vec{\boldsymbol{v}}_{\text{avg}} = \frac{1}{2}\left(\vec{\boldsymbol{v}}_i + \vec{\boldsymbol{v}}_f\right)$, $\quad v_f^2 = v_i^2 + 2\vec{\boldsymbol{a}} \cdot (\vec{\boldsymbol{r}}_f - \vec{\boldsymbol{r}}_i)$,

Relative velocities: $\quad \vec{\boldsymbol{v}}_{CA} = \vec{\boldsymbol{v}}_{CB} + \vec{\boldsymbol{v}}_{BA}$

Circular motion: (angles in radians)



$s = r\theta$ $\omega = \frac{d\theta}{dt}$ $v = r\omega$ $a_c = \frac{v^2}{r} = r\omega^2$

Uniform: $v = \frac{2\pi r}{t}$

Nonuniform: $a_t = \frac{dv}{dt}$, $a_r = \frac{v^2}{r}$.

Forces

$\vec{F}_{\text{net}} = m\vec{a}$ $\vec{F}_{AB} = -\vec{F}_{BA}$ Elasticity (Hooke's Law): $F = -kx$

Weight: $F_g = mg$, $g = 9.8\ \text{m/s}^2$ on earth. Friction: $F < \mu_s F_N$ or $F = \mu_k F_N$.

Work and Energy

$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ or $W = \vec{F} \cdot \Delta\vec{r}$. 1 dimension: $W = \int F(x)dx$ or $W = F\Delta x$

$K = \frac{1}{2}mv^2$ $\Delta K = W$ $\Delta U = -W$ $E = K + U$ $F = -\frac{dU}{dx}$

$\Delta E = W_{nc}$ $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$ Elasticity (Hooke's Law): $U = \frac{1}{2}kx^2$

Gravity (near earth): $U = mgh$ with $U = 0$ at $h = 0$.

Momentum, Impulse, and Center of Mass Motion

$$\vec{p} = m\vec{v} \qquad \vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{I} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$$

$$\vec{x}_{\text{cm}} = \frac{1}{M_{\text{total}}}\int \vec{x}\,dm \qquad \sum \vec{p} = M_{\text{total}}\vec{v}_{\text{cm}} \qquad \sum \vec{F} = M_{\text{total}}\vec{a}_{\text{cm}}$$

Collisions: $\sum \vec{p}_i = \sum \vec{p}_f$ Elastic collisions: $\sum K_i = \sum K_f$, 1 dim: $v_2^i - v_1^i = v_1^f - v_2^f$

Rigid Body Motion

Substitututions: (linear quantity →angular quantity)

$$x \to \theta, \quad v \to \omega, \quad a \to \alpha, \quad m \to I, \quad \vec{\boldsymbol{F}} \to \vec{\boldsymbol{\tau}}, \quad \vec{\boldsymbol{p}} \to \vec{\boldsymbol{L}}$$

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ Rolling: $v = r\omega\,,\ a = r\alpha$

$I = \int r^2 dm$ $K_{\text{rot}} = \frac{1}{2} I\omega^2$ $K = K_{\text{cm}} + K_{\text{rot}}$ $L = I\omega$

$\tau = rF\sin\theta$ $\tau = I\alpha$ $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}$ $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$ $\vec{\boldsymbol{\tau}} = \frac{d\vec{\boldsymbol{L}}}{dt}$

Fluids

Pressure: $P = \frac{F}{A}$ $P = P_0 + \rho g h$ Atmospheric pressure: $1\ \text{Atm} = 1.013 \times 10^5\ \text{N/m}^2$

Buoyancy: $F_B = \rho_{\text{fluid}}\, gV.$ Density of water: $\rho_{\text{w}} = 1\ \text{g/cm}^3 = 1000\ \text{kg/m}^3$

Volume rate of flow: $Av =$ constant

Bernoulli equation: $P + \frac{1}{2}\rho v^2 + \rho g h =$ constant

Oscillation

$$F = -kx, \quad a = -\frac{k}{m}x, \quad x = A\cos(\omega t + \phi_0), \quad v_{\max} = \omega A, \quad a_{\max} = \omega^2 A$$

$\omega = \sqrt{\frac{k}{m}}$, $T = \frac{2\pi}{\omega}$, $K = \frac{1}{2}mv^2$, $U = \frac{1}{2}kx^2$, $E = K + U =$ constant

Pendulum: (small amplitude) effective $k \approx \frac{mg}{L}$, $\omega = \sqrt{g/L}$.

Units

$1\ \text{N} = 1\ \text{kg m/s}^2$ $1\ \text{J} = 1\ \text{Nm}$ $1\ \text{W} = 1\ \text{J/s}$ $1\ \text{h.p.} = 746\ \text{W}$

$1\ \text{degree} = \frac{\pi}{180}\ \text{rad}$ $1\ \text{Pa} = 1\ \text{N/m}^2$