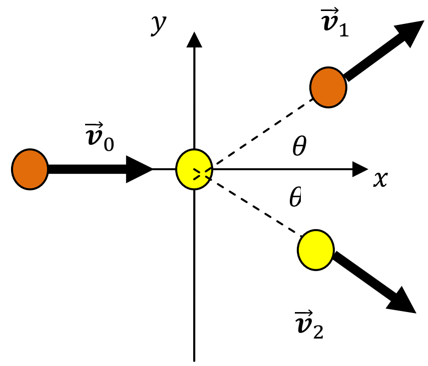
PHYSICS 221 EXAM 3

November 16, 2009

Solutions

Problem 1: [17pt]

Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in a glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed of 8.00 m/s. After the collision, both disks move along directions that make an angle of $\theta = 36.9°$ with the orange disk's initial direction of motion, as shown.



(a) [6pt] If the mass of each disk is m, and the final speeds of the orange an yellow puck are v_1, v_2, explicitly write the two equations expressing momentum conservation in the x and y directions. Use θ for both angles, not the numerical values. Write one equation for each direction. Do not combine them or skip steps, even if they seem obvious to you.)

$$x: \quad mv_0 = mv_1 \cos\theta + m\, v_2 \cos\theta$$
$$y: \quad 0 = mv_1 \sin\theta - m\, v_2 \sin\theta.$$

(b) [6pt] Calculate the final speed of each disk.

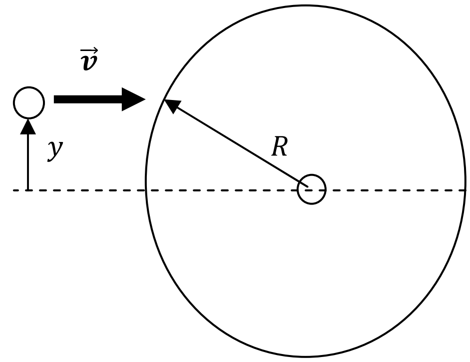
The y equation implies that $v_1 = v_2$, so the x equation reduces to $v_0 = 2v_1 \cos\theta$. Since $v_0 = 8.0 \text{ m/s}$ and $\cos 36.9° = 0.800$, we find that $\boxed{v_1 = v_2 = 5.00 \text{ m/s.}}$

(c) [5pt] How much (if any) energy was converted to internal energy (spinning, heat, vibrations, …) in the collision if $m = 0.400$ kg?

The energy converted to other forms is

$$\Delta E = \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}(0.400 \text{ kg})(8^2 - 2 \times 5^2) \text{ m}^2/\text{s}^2 = \boxed{2.80 \text{ J.}}$$

Problem 2: [17pt] A cannonball of mass $m = 200$ kg is fired at a velocity of 125 m/s toward a large wooden disk of mass $M = 500$ kg with a radius of $R =$ 1.20 m, mounted on a central axis about which it is free to spin. The cannonball gets embedded in the rim a height $y =$ 0.30 m above the axis, and the wheel starts to spin with the cannonball attached.



(a) [5pt] What is the magnitude and direction of the cannonball's angular momentum **vector** about the axis of the wheel just before it strikes?

$\vec{\boldsymbol{L}} = \vec{\boldsymbol{R}} \times m\vec{\boldsymbol{v}} =$ $\boxed{mvy = 7500 \text{ kg m}^2/\text{s into the page}}$, using the right-hand rule.

(Either the symbolic or numerical form gets credit, according to the instructions in class.)

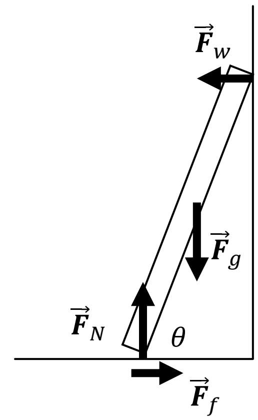
(b) [6pt] If the moment of inertia of the wheel about its axis is $I = \frac{1}{2}MR^2$, what is the moment of inertia of the wheel with the embedded cannonball? Give a numerical result.

$$I = \frac{1}{2}MR^2 + mR^2 = 450 \text{ kg} \times (1.2 \text{ m})^2 = \boxed{648 \text{ kg} \cdot \text{m}^2}$$

(c) [6pt] What is the angular velocity of the wheel with the embedded cannonball after the collision?

Conservation of angular momentum implies that $L = 7500\ kg\frac{m^2}{s} = I\omega$, giving $\omega =$ $\boxed{11.6 \text{ rad/s.}}$

Problem 3: [10pt] A uniform ladder of length $L = 15.0$ m and mass $m = 10.0$ kg rests against a frictionless wall. The floor is not frictionless, however. The ladder can be set at an angle of at most $\theta = 60°$ before it starts to slip.



(a) [4pt] What is the torque due to **each** of the four forces shown in the diagram about the bottom edge of the ladder? Express it in terms of the symbolic magnitudes of the forces, the length L and the angle θ. (Do not use the numerical values in this part.) Use the convention that counterclockwise torques are positive. Do not add the torques.

The torque is the force time the lever arm from the bottom of the ladder. This gives the torques for each of the four forces,

$\vec{F}_f$: 0,	$\vec{F}_N$: 0,	$\vec{F}_g$: $-\frac{1}{2}LF_g \cos\theta$,	$\vec{F}_w$: $LF_w \sin\theta$

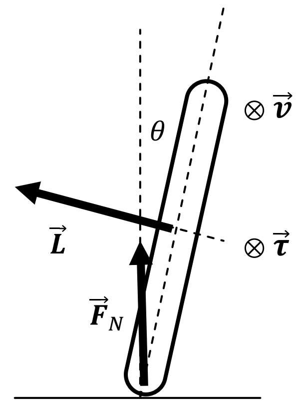
with the sign convention that counterclockwise torques are positive.

(b) [6pt] Find the coefficient of friction of the ladder against the floor.

Balancing torque about the bottom of the ladder gives $\frac{1}{2}F_g \cos\theta = F_w \sin\theta$ and balancing forces gives $F_N = F_g = 2\,F_w \tan\theta = 2\,F_f \tan\theta$, so

$$\mu_s = \frac{F_f}{F_N} = \frac{1}{2\tan\theta} = \frac{1}{2\sqrt{3}} = \boxed{0.289.}$$

Problem 4. [6pt] A wheel is rolling away from you as shown in the figure. It is tilted toward the right as it rolls away. (The velocity vector is into the page.)



(a) [2pt] What is the direction of its angular momentum vector?

(You can draw on the figure if it helps explain your answers.)

It is in the direction of your thumb when you curl your fingers along the rim of the wheel as it turns. The direction is left along the axle.

(b) [2pt] What is the direction of the net torque vector acting on the wheel about its center of mass?

The torque is due to the normal force acting upward on the bottom of the wheel, which creates a clockwise torque about the center of the wheel. Curling your fingers clockwise, your thumb points into the page, in the same direction as $\vec{v}$.

(c) [2pt] Explain briefly in words what effect this torque has on the motion of the wheel. Be specific about any directions involved.

The torque causes a change $\Delta\vec{L} = \vec{\tau}\,\Delta t$ in the angular momentum that is into the page, parallel to the torque. Since this is perpendicular to $\vec{L}$, the magnitude of $\vec{L}$ doesn't change, but the direction does, rotating away from you from the left. It follows the axle of the wheel, so the entire wheel steers toward the right as it rolls away.

View from above: