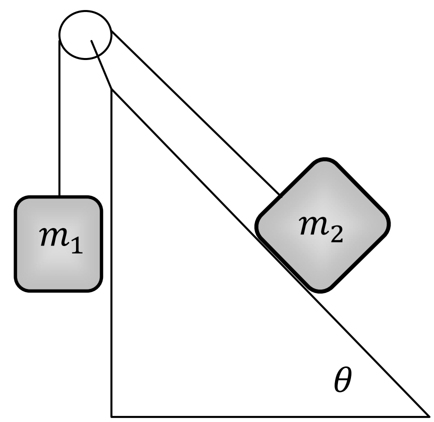
# PHYSICS 221 EXAM 2

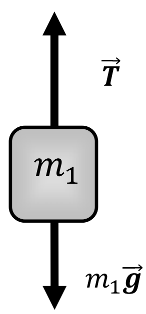
## October 14, 2009

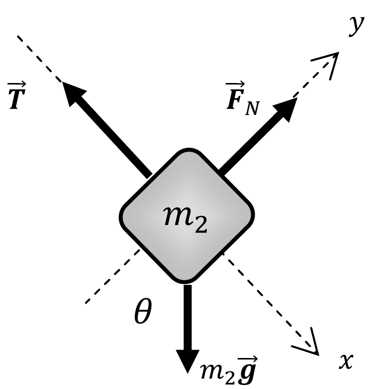
## Solutions

**Problem 1: [22pt]** Two blocks of mass $m_1 = 2.0$ kg and $m_2 = 6.0$ kg are connected by a rope of negligible mass, with $m_2$ sliding on a frictionless plane tilted upward at angle $\theta = 53°$.




**(a) [5pt]** Draw a free body diagram for each block, showing and labeling all forces acting on the block.



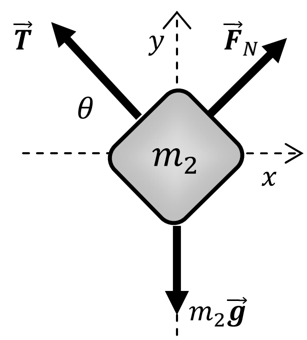




**(b) [5pt]** Write Newton's Law for the motion of each block, relating the forces to the acceleration. You should have an equation for each block. For definiteness, take the acceleration to be positive if $m_1$ is rising.

Block 1: $\boxed{T - m_1 g = m_1 a\ .}$

Block 2: In the $x$ direction, which is taken to be along the plane, $\boxed{m_2 g \sin\theta - T = m_2 a.}$ In the $y$ direction, forces balance, so that $F_N = m_2 g \cos\theta$, but the normal force is not needed to find the acceleration.

Alternatively, you could take the $x$ direction to be horizontal and the $y$ direction to be vertical for block 2. If so, you would need two equations, since there are two nonzero components of the acceleration:




$$T\sin\theta + F_N\cos\theta - m_2 g = m_2 a_y\,,$$

$$F_N\sin\theta - T\cos\theta = m_2 a_x.$$

**(c) [6pt]** Determine the acceleration of the blocks. Clearly note whether block 1 is rising or falling.

Adding the equations for blocks 1 and 2 obtained with the tilted axes gives

$$m_2 g\sin\theta - m_1 g = (m_1 + m_2)a.$$

Then

$$a = \frac{m_2\sin\theta - m_1}{m_1 + m_2}g = \frac{6\sin 53^\circ - 2}{8}g = 0.349\,g = \boxed{3.42\,\frac{\text{m}}{\text{s}^2}.}$$

If you didn't tilt the axes, then the components of the acceleration for block 2 are related to the acceleration of block 1 via $a_x = a\cos\theta, a_y = -a\sin\theta$. Substituting these into the equations obtained with this choice of axes gives

$$T\sin\theta + F_N\cos\theta - m_2 g = -m_2 a\sin\theta\,,$$
$$F_N\sin\theta - T\cos\theta = m_2 a\cos\theta.$$

The normal force can be eliminated by multiplying the top equation by $-\sin\theta$, the bottom equation by $\cos\theta$, and then adding:

$$\begin{array}{rl} m_2 g\sin\theta - T\sin^2\theta - F_N\sin\theta\cos\theta &= -m_2 a\sin^2\theta \\ -T\cos^2\theta + F_N\sin\theta\cos\theta &= \quad m_2 a\cos^2\theta \\ \hline m_2 g\sin\theta - T(\sin^2\theta + \cos^2\theta) &= m_2 a\,(\sin^2\theta + \cos^2\theta). \end{array}$$

Using the fact that $\sin^2\theta + \cos^2\theta = 1$ gives $m_2 g\sin\theta - T = m_2 a$, which can then be combined with the equation for block 1 as above to obtain the acceleration $a$.

**(d) [5pt]** Find the tension in the string.

From the equation for block 1,

$$T = m_1(g + a) = 1.349\, m_1 g = 1.349\,(2\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right) = \boxed{26.44\text{ N.}}$$

**(e) [5pt]** Find the net power applied to mass $m_1$ by the forces acting on it at a time $t = 2.0$ s after the blocks are released from rest.

The power is $P = Fv = m_1 a v$, with $v = at = 6.84\frac{m}{s}$. Then

$$P = (2\text{ kg})\left(3.42\frac{\text{m}}{\text{s}^2}\right)\left(6.84\frac{\text{m}}{\text{s}}\right) = \boxed{46.8\text{ W.}}$$

Alternatively, $P = \frac{dW}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}m_1 v^2\right) = m_1 v\frac{dv}{dt} = m_1 v a$, which gives the same result. This method uses the work-energy theorem, $W = \Delta K$ in the second step.

Note that the power is applied by both tension and gravity. The power due to the tension alone is $P_T = Tv = (26.44\, N)\left(6.84\frac{m}{s}\right) = 180.9\, W$, while the power due to gravity is $P_g = -m_1 g v = -134.1\, W$, with total $P = P_T + P_g = 46.8\, W$. The power applied by tension alone can also be written as $P_T = \frac{dE}{dt}$ if $E = K + m_1 g y$ is the sum of the kinetic and gravitational potential energy of block 1, since the tension is working against gravity.

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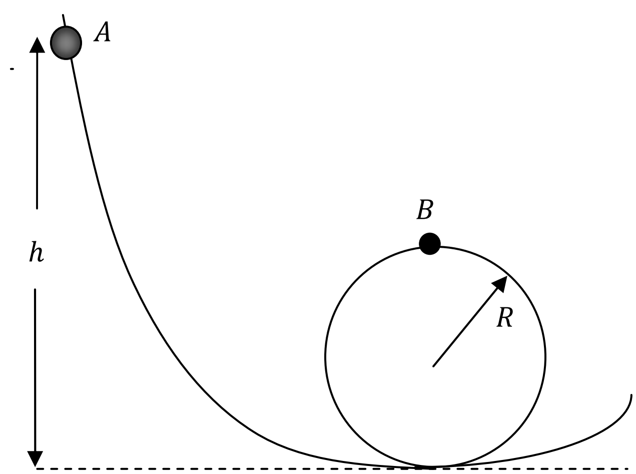
**Problem 2: [18pt]** A bead of mass $m$ slides without friction around a wire bent into the shape shown. The bead is released from rest at point $A$, at a height $h = 4R$, where $R$ is the radius of the loop.

This is a symbolic problem, and should not require the use of a calculator.



**(a) [5pt]** What is the bead's potential energy at each of the points $A$ and $B$? Express the results in terms of $m$, $R$ and $g$. Assume the potential energy is zero on the dashed line at the bottom of the figure.

At point $A$, $U_A = mgh = \boxed{4mgR}$. At point $B$, $U_B = \boxed{2mgR}$.

**(b) [5pt]** What is the bead's kinetic energy at point $B$? Express the result in terms of $m$, $R$ and $g$.

Energy conservation implies that $U_A = U_B + K_B$, so $K_B = U_A - U_B = \boxed{2mgR}$.

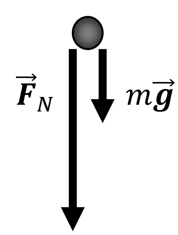
**(c) [5pt]** What is the bead's acceleration at point $B$? Give the magnitude and direction. Express the result in terms of $g$.

The acceleration is centripetal, directed downward. The magnitude is given by $a = \frac{v^2}{R}$. The kinetic energy is $K_B = \frac{1}{2}mv^2 = 2mgR$, so $v^2 = 4gR$, giving

$$a = \boxed{4g}.$$

**(d) [5pt]** What is the normal force on the bead at point $B$? Is it up or down? Express the result in term of the weight $mg$ of the bead.

The free body diagram for the bead at point $B$ is shown at the right. By Newton's Law, $F_N + mg = ma = 4mg$, so that the normal force is




$$F_N = \boxed{3mg}$$

directed $\boxed{\text{downward}}$, so that it adds to the weight in the net force.

**(e) [5pt]** Suppose there is friction in the system, so that when it is released from point $A$, it just makes it to point $B$ and stops. How much work was done by friction along the wire? Express the result in terms of $m, g$, and $R$.

The initial total energy was $E_A = U_A = 4mgR$, and the final energy was $E_B = U_B = 2mgR$, so the work done by friction was $W_{nc} = E_B - E_A = \boxed{-2mgR}$.