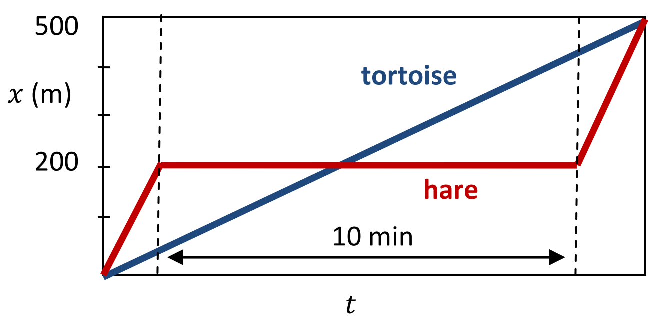
PHYSICS 221 EXAM 1

September 16, 2009

Solutions

Problem 1: [17pt] A tortoise and a hare are running a 500 m race The hare has a comfortable lead, and stops to rest after 200 m. At this time, the tortoise has crawled only 50 m, but is continuing toward the finish line at a steady pace. The hare waits 10 minutes to let the tortoise get ahead, and then starts running at its original pace, just in time to finish in a tie.

(a) [5pt] Draw a position-time diagram for the race, showing both animals.



(b) [**6pt]** How long did the entire race take?

Let t be the time for the entire race, and v_t, v_h be the speeds of the tortoise and hare. Then $500\text{ m} = v_t t = v_h(t - 10\text{ min})$. Since the hare ran 200 m while the tortoise crawled 50 m, $v_h = 4\,v_t$. Therefore, $v_t t = 4v_t(t - 10\text{ min})$ and $3t = 40\text{ min}$, giving t = **13.33 min** = **800 seconds**.

(c) [6pt] What were the speeds of the tortoise and the hare in m/s?

$$v_t = \frac{500\ m}{800\ s} = \mathbf{0.625\frac{m}{s}}, \quad v_h = 4v_t = \mathbf{2.5\frac{m}{s}}.$$

Problem 2: [16pt] A place kicker must kick a football from a point 36 m from the goal. He kicks the ball upward at an angle of 45°, sending the ball high enough to pass comfortably through the goal posts at a height of 4.8 m above the ground.

(a) [5pt] What was the initial speed of the football just after it was kicked?

Let x be the distance to the goal post y be the height of the ball there, and t be the time to reach this position. Then

$$x = v_x t, \quad y = v_y t - \frac{1}{2}gt^2 = x - \frac{1}{2}gt^2$$

since $v_{y0} = v_x$ at 45°. Then $t^2 = \frac{2}{g}(x - y) = \frac{2}{9.8}(31.2)\ \frac{\mathrm{m}}{\mathrm{s}^2} = 6.37\frac{\mathrm{m}}{\mathrm{s}^2}$ and $t = 2.52\ s$. Then $v_x = \frac{x}{t} = 14.3\frac{\mathrm{m}}{\mathrm{s}} = v_0 \cos\theta = \frac{v_0}{\sqrt{2}}$ and $v_0 = \mathbf{20.2\frac{m}{s}}$.

(b) [5pt] How long was the football in the air before it hit the ground again?

The time the ball went up is $t_{up} = \frac{v_{0y}}{g} = \frac{v_x}{g} = \frac{14.3}{9.8}\mathrm{s} = 1.46\ \mathrm{s}$. The entire time in the air is twice this, $\mathbf{2.92\ s}$.

(c) [6pt] What was the football's velocity as it passed over the goal post? Give the speed and direction angle relative to the horizontal, with positive angle upward, negative downward. [If you don't have an answer to part (a), you may use $v_0 = 22\frac{\mathrm{m}}{\mathrm{s}}$.]

The velocity components are

$$v_x = 14.3\frac{\mathrm{m}}{\mathrm{s}}, \qquad v_y = v_{y0} - gt = 14.3\frac{\mathrm{m}}{\mathrm{s}} - \left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(2.52\ \mathrm{s}) = -10.4\frac{\mathrm{m}}{\mathrm{s}}.$$

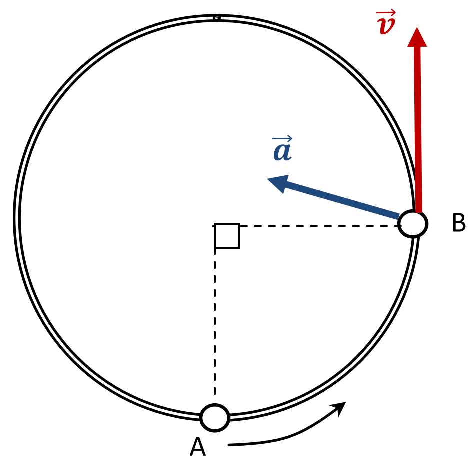
Therefore, the speed of the ball over the goal posts is

$$v = \sqrt{v_x^2 + v_y^2} = \mathbf{17.7\frac{m}{s}}$$

and the angle with respect to horizontal is

$$\theta = \mathrm{atan}\left(\frac{-10.4}{14.3}\right) = \mathrm{atan}(-0.727) = \mathbf{-36.0°}.$$

Problem 3: [17pt] A car drives around a circular track 1200 m in circumference, with constant tangential acceleration, starting from rest at point A, travelling counter-clockwise, and completing one revolution in 52 seconds.



(a) [5pt] What is the car's tangential acceleration?

This is the acceleration in the direction of motion. It can be found using the equation

$$1200\ \text{m} = \frac{1}{2} a_t t^2 = (1352\ \text{s}^2)\ a_t,$$

$$a_t = \mathbf{0.888 \frac{m}{s^2}}.$$

(b) [6pt] What is the car's velocity vector at point B, which is ¼ of the way around the track? Give the magnitude and angle relative to the forward direction.

The velocity vector points in the direction of motion and has a magnitude equal to the speed. The speed can be found from the equation $v^2 = 2ad$, with $d = 300$ m for ¼ of the way around the track. Therefore, $v^2 = 2(0.888)(300) \frac{\text{m}^2}{\text{s}^2} = 532.5 \frac{\text{m}^2}{\text{s}^2}$ and $v = \mathbf{23.1 \frac{m}{s}}$. The angle relative to the forward direction is $\mathbf{0°}$.

(c) [6pt] What is the car's total acceleration vector at point B? Give the magnitude and angle relative to the forward direction. [If you don't have answers to the previous parts, you may use $a_t = 1.0 \frac{m}{s^2}, v = 20 \frac{m}{s}$.]

The component of the acceleration in the forward direction is $a_t = 0.888 \frac{\text{m}}{\text{s}^2}$. The centripetal, or radial, acceleration is $a_r = \frac{v^2}{R}$ with $v^2 = 532.5 \frac{\text{m}^2}{\text{s}^2}$ and $R = \frac{1200\ \text{m}}{2\pi} = 191$ m, giving $a_r = 2.74 \frac{\text{m}}{\text{s}^2}$, directed toward the center (left).

Therefore, the magnitude and direction of the acceleration are

$$a = \sqrt{a_t^2 + a_r^2} = \mathbf{2.88 \frac{m}{s^2}}, \quad \theta = \text{atan}\left(\frac{2.74}{0.888}\right) = \text{atan}(3.09) = \mathbf{72.0}°$$

left of the forward direction.