



Bhabha Process at LEP - Theoretical Calculations*

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In this contribution we give a short overview of the situation in the precision calculation of the Bhabha process and we present a preliminary numerical result on the second-order sub-leading correction to the small angle Bhabha process.

1. Introduction

The Bhabha scattering process consists in fact of two distinct processes (especially at the Z peak): one is the Small Angle Bhabha (SABH) process below about 6° in the scattering angle, which is dominated by the gamma t -channel exchange and another one, the Large Angle Bhabha (LABH) process above 6° , which gets important contributions from various s -channel (annihilation) exchanges. The SABH process is employed almost exclusively to determine the luminosity in the e^+e^- colliders, using dedicated luminometer sub-detectors. The LABH provides input data for precision electroweak tests of the Standard Model (SM), in particular the electron partial width Γ_e of the Z boson. In this presentation we shall concentrate mainly on the SABH process. At LEP at $\sqrt{s} = M_Z$, in the 1° – 3° angular range it gives about four times more events than Z decays. It is therefore ideally suited for precise measurements of the luminosity from the point of view of statistical error. Even more important, it is dominated by “known physics”, that is by t -channel exchange of a photon. It is therefore calculable from “first principles”, i.e. from the Lagrangian of the Quantum Electrodynamics (QED) with the standard Quantum Field Theoretic methods, Feynman diagrams, etc.

1.1. Theoretical error in the luminosity measurement

At present, the luminosity measurement at LEP using the SABH process has a very small statistical and experimental systematic error, typically $0.07 - 0.15\%$. The uncertainty of the theoretical calculation of the SABH process must be combined with this error. It is called the “theoretical error” (the theory uncertainty) of the luminosity. It was last year reduced to 0.16% [1] and is now at the level of 0.11% [2, 3]: in spite of the progress, it is still a dominant component of the total luminosity error. This error enters into that of the total cross section measured at LEP. The experimental precision of the so-called invisible width (number of neutrinos) is strongly dependent on the precision of the luminosity measurement. The other quantities used for tests of the SM are also affected. In Table 1 we show the influence of the luminosity error on the LEP measurable used in the test of the SM.

Obviously it would be worthwhile to lower the theoretical uncertainty in the calculation of the SABH cross section below the future, ultimate, experimental precision of LEP experiments, which will probably reach 0.05% . From the beginning of the LEP operation both experimental and theoretical components in the error of the luminosity went from the level of 2% to 0.1% . Why has it been difficult to reduce the theoretical error even further? The main obstacles were the need for non-trivial calculations of the higher-order contributions and the complicated Event

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	Theoretical luminosity error		
	0.16%	0.11%	0.06%
m_Z [GeV]	91.1884 ± 0.0022	91.1884 ± 0.0022	91.1884 ± 0.0022
Γ_Z [GeV]	2.4962 ± 0.0032	2.4962 ± 0.0032	2.4961 ± 0.0032
σ_h^0 [nb]	41.487 ± 0.075	41.487 ± 0.057	41.487 ± 0.044
R_l	20.788 ± 0.032	20.787 ± 0.032	20.786 ± 0.032
$A_{FB}^{0,l}$	0.0173 ± 0.0012	0.0173 ± 0.0012	0.0173 ± 0.0012
Γ_{had} [GeV]	1.7447 ± 0.0030	1.7447 ± 0.0028	1.7446 ± 0.0027
Γ_{ll} [MeV]	83.93 ± 0.13	83.93 ± 0.13	83.93 ± 0.12
σ_{ll}^0 [nb]	1.9957 ± 0.0044	1.9958 ± 0.0038	1.9959 ± 0.0034
Γ_{had}/Γ_Z [%]	69.90 ± 0.089	69.90 ± 0.079	69.89 ± 0.072
Γ_{ll}/Γ_Z [%]	3.362 ± 0.0037	3.362 ± 0.0032	3.362 ± 0.0028
Γ_{inv} [MeV]	499.9 ± 2.4	499.9 ± 2.1	499.9 ± 1.9
Γ_{inv}/Γ_{ll} [%]	5.956 ± 0.030	5.956 ± 0.024	5.956 ± 0.020
N_ν	2.990 ± 0.015	2.990 ± 0.013	2.990 ± 0.011

Table 1

Line shape and asymmetry parameters from 5-parameter fits to the data of the four LEP1 experiments, made with a theoretical luminosity error of 0.16%, 0.11% and 0.06% [4]. In the lower part of the table also derived parameters are listed.

Selection (ES) in the actual measurement. Due to the complicated ES, the phase-space boundaries in the calculation of the SABH cross section are too complicated for any analytical calculation. The calculation must be numerical, the best being in the form of the Monte Carlo event generator MCEG. The theoretical calculation would be completely useless if in the calculation of the SABH cross section we did not control its “technical precision”, corresponding to *all* possible numerical uncertainties. The control over the technical precision is probably the most difficult and labour-consuming part of the enterprise.

1.2. BHLUMI MC event generator

In the recent years the LEP collaborations have used the BHLUMI Monte Carlo event generator to calculate the SABH cross section for any type of experimental ES. The program, originally written in 1988 [5], was published in 1992 [6] with the first-order QED matrix element $\mathcal{O}(\alpha^1)_{exp}$ (exponentiation according to the Yennie-Frautschi-Suura theory) and its matrix element was recently upgraded by means of adding the missing second-order in the Leading-Logarithmic approximation [7]. BHLUMI provides multiple soft and hard photons in the complete phase-

space in all versions. The multi-photon integration over multi-photon phase-space remains essentially unchanged in BHLUMI since the first version. Gradual improvements concern mainly the matrix element. An increasing number of cross-checks are built up in order to determine better its technical precision, see [1, 8].

1.3. Importance of the various QED corrections

The electron mass is very small and the Leading-Logarithmic approximation in terms of the large logarithm $L = \ln(|t|/m_e^2) - 1$ is a very useful tool. In Table 2 we show numerical values of the “canonical coefficients” for various LL and sub-leading QED radiative corrections. As we see from the table, for a precision of order 0.5% it is enough to include the entire first-order $\mathcal{O}(\alpha)$ and the second-order leading-log $\mathcal{O}(\alpha^2 L^2)$, while at the present precision, of order 0.05%–0.10%, it is necessary to have control over $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^3)$.

1.4. Outline

In the following section we shall briefly summarize the results of the Bhabha Working group in the recent LEP2 workshop, which led to a new lower precision estimate of 0.11% of the theoret-

		$\theta_{min} = 30 \text{ mrad}$		$\theta_{min} = 60 \text{ mrad}$	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi} 4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2} \frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2 L^2)$	$\frac{1}{2} \left(\frac{\alpha}{\pi} 4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2 L)$	$\frac{\alpha}{\pi} \left(\frac{\alpha}{\pi} 4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3 L^3)$	$\frac{1}{3!} \left(\frac{\alpha}{\pi} 4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

Table 2

The canonical coefficients indicating the generic magnitude of various leading and sub-leading contributions up to third-order. The large-log $L = \ln(|t|/m_e^2) - 1$ is calculated for $\theta_{min} = 30 \text{ mrad}$ and $\theta_{min} = 60 \text{ mrad}$ and for two values of the centre of mass energy: at LEP1 ($\sqrt{s} = M_Z$), where the corresponding values of $|t| = (s/4)\theta_{min}^2$ are 1.86 and 7.53 GeV^2 , and at a LEP2 energy ($\sqrt{s} = 200 \text{ GeV}$), where the corresponding value of $|t|$ are 9 and 36 GeV^2 , respectively.

ical uncertainty in the luminosity. In the next section we shall give a glimpse of the recent calculation of the critical second-order sub-leading correction to the SABH process, which opens the route towards a theoretical precision below 0.10%.

2. Bhabha Working group of LEP2 workshop

The main aim of the 1995 Bhabha Working group was to compare different QED calculations for the SABH and LABH processes, in order to verify and/or improve precision estimates for these calculations, for both LEP1 and LEP2 applications. It must be stressed that it was really the first systematic and organized example of such a comparison, although the first step in this direction was already taken in Ref. [9]. The QED calculations for the SABH process, apart from BHLUMI, were provided by four groups of authors: the names of the SABH programs/calculations are² SABSPV [10], BHAGEN95 [11] and NLLBHA [12]. Among these four, BHLUMI represents a full-scale event generator, SABSPV and BHAGEN95 are MC pro-

grams providing the total cross section for arbitrary ES, and NLLBHA is a semi-analytical program able to calculate the total cross section only for certain special (unrealistic) ES's. The QED matrix element in all the calculations includes complete first-order and second-order in the leading-logarithmic approximation, with the notable exception of NLLBHA, which features in addition, the second-order next-to-leading-log corrections. All calculations feature some kind of exponentiation and part or all of the third-order LL corrections.

To understand better the differences between the four calculations the WG participants agreed on four examples of the “standard event selection” (SES) named BARE1, CALO1, CALO2, SICAL2. These vary from the simple and unrealistic BARE1 (for which NLLBHA can provide results) to the more sophisticated CALO1 and CALO2, ending with SICAL2, which is very close to a typical experimental ES at LEP. All four SES's, represent the “double tag” type of event selection, that is both e^+ and e^- have to be observed in the forward/backward direction with a certain minimum energy and minimum scattering angle. For BARE1 the observed scattered objects are just the “bare” e^+ and e^- while accompanying bremsstrahlung photons are ignored

²Here we gave only one reference per program, see [2] for an exhaustive list of relevant references.

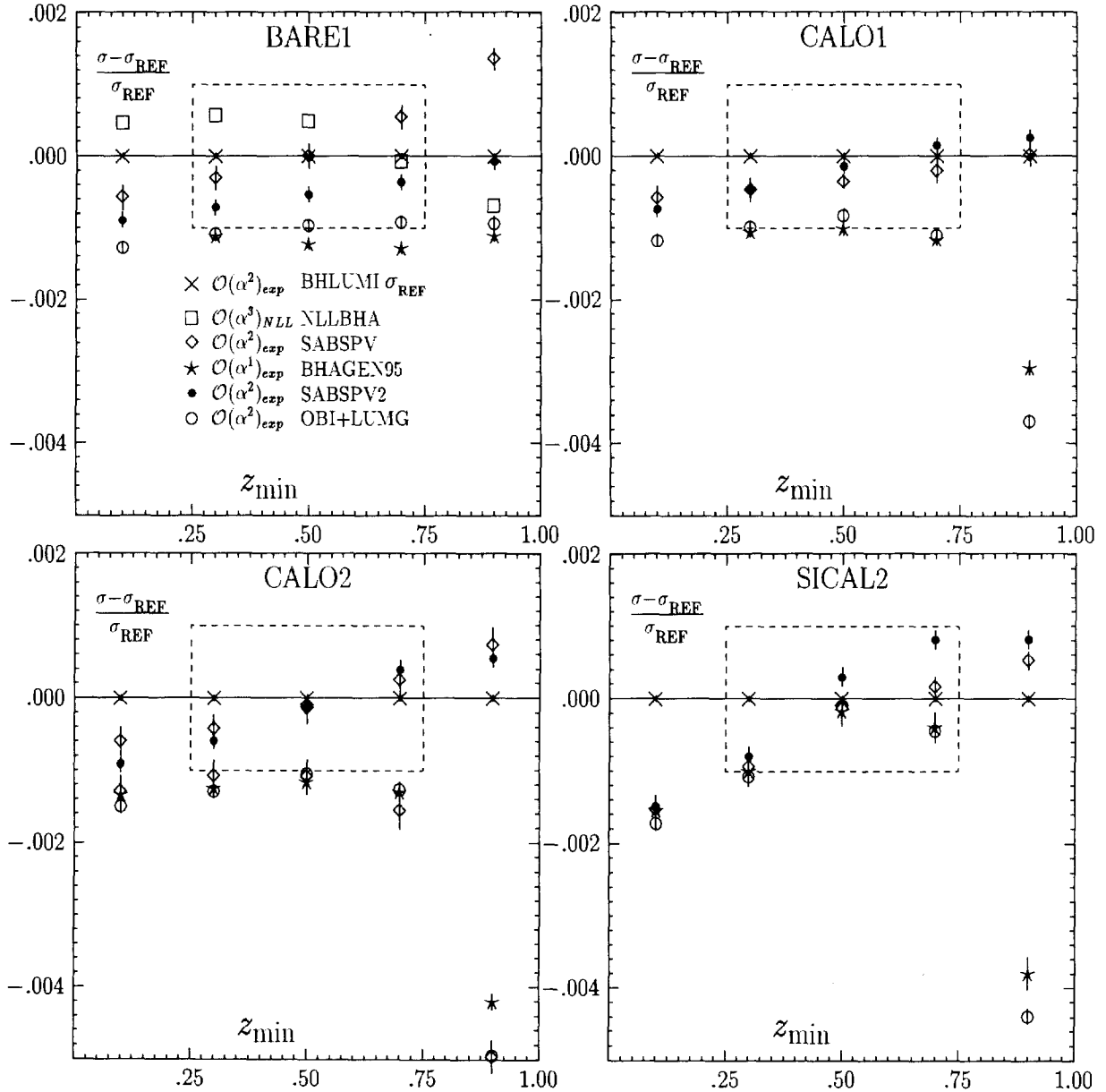


Figure 1. Monte Carlo results for the symmetric Wide-Wide ES's BARE1, CALO1, CALO2 and SICAL2, for matrix elements beyond first-order. Z exchange, up-down interference and vacuum polarisation are switched off. The centre of mass energy is $\sqrt{s} = 92.3$ GeV. In the plot, the $\mathcal{O}(\alpha^2)_{exp}^{YFS}$ cross section σ_{BHL} from BHLUMI 4.02.a is used as a reference cross section.

completely. This is unrealistic because all LEP luminometers are of the calorimetric type, i.e. they combine in the final state the e^\pm with the pho-

tons that are close to them into single objects, "clusters". The scattering angle of the cluster (which is the angle of its centre) is required to be

above a certain minimum angle, below a certain maximum angle, and the total energy of the cluster must be above a certain threshold, typically half of the beam energy. Note that the most sophisticated silicon luminometers are not able to distinguish electrons and photons at all! All three SES's, CALO1, CALO2 and SICAL2, are calorimetric and differ in the way the cluster is defined. CALO1 uses an angular cone around the directions of e^\pm in order to associate photons with the e^\pm while CALO2 defines the cluster in terms of a plaquette in the (θ, ϕ) plane centred around e^\pm . CALO2 and CALO1 still make a distinction between electrons and photons. SICAL2 is completely charge-blind and forms a cluster around the most energetic e^\pm or photon exactly as in the silicon luminometer of ALEPH or OPAL. Finally, let us note that all four ES's are in two versions, "symmetric" and "asymmetric". In the symmetric version the minimum and maximum value of the scattering angle θ in the forward and backward hemisphere are the same; in the asymmetric version, they are not. The real experimental ES is asymmetric in order to eliminate effects due to the geometrical uncertainty of the interaction point. The interested reader will find in Ref. [2] a more detailed description of the above four ES's.

The comparisons between the four calculations started with the warming-up exercise in which all four groups have calculated *the same* first-order $\mathcal{O}(\alpha^1)$ cross for all four SES's (except NLLBHA, which is able to provide a cross section only for BARE1). After some adjustment of the matrix element (Z-exchange and vacuum polarization were switched off) and debugging of the programs for SES's very good agreement was obtained. The cross sections at $\mathcal{O}(\alpha^1)$ agreed to within 0.03%. The calibration test was passed successfully for all SES's including the realistic SICAL2. In the above test the matrix element was exactly the same and what was really seen as a difference was the pure technical precision. It would be ideal to extend this kind of test to second-order, but here we could not do the same. The matrix elements are not compatible, because in most of the calculations the second-order sub-leading terms are incomplete. The procedure of the phase-space integration over the two real photons is for SAB-

SPV and BHAGEN95 inherently tied up with adding second-order LL correction and/or exponentiation.

In the next step the comparison of all calculations was attempted for all four SES's for the matrix element "beyond-first-order", using the best available QED matrix element for a given program. In order to minimize the possible differences, Z-exchange and vacuum polarization were temporarily switched off. The results of the calculations are shown in Fig. 1. All cross sections are compared with the BHLUMI cross section, which is used as a reference³. The differences with BHLUMI are plotted as a function of the dimensionless energy-cut parameter z_{\min} . The minimum energy of e^\pm in units of beam energy is approximately z_{\min} , and the actual definition of z_{\min} is slightly different for each ES, see Ref. [2] for more details.

As we see in Fig. 1, we have included in the comparisons another cross section calculated by means of BHLUMI, which is referred to as being computed by OLDBIS + LUMLOG. The corresponding method of adding the second-order LL correction to the first-order cross section (for arbitrary ES) was described in Ref. [13] and the tools to calculate it are included in the BHLUMI as separate sub-programs. These tools are: (a) the first-order event generator OLDBIS and (b) the LL MC event generator LUMLOG, which generates photons in the strictly collinear approximation. The comparison of BHLUMI with OLDBIS + LUMLOG was used in Refs. [14, 1] in order to estimate the technical precision of BHLUMI and the missing higher-order and/or sub-leading contributions. In fact the OLDBIS + LUMLOG recipe is quite similar to the SABSPV calculation. The main difference is that while the OLDBIS + LUMLOG prescription for combining $\mathcal{O}(\alpha)$ cross section with the $\mathcal{O}(\alpha^2)_{\text{LL}}$ was "additive"

$$\mathcal{O}(\alpha^1) + \{\mathcal{O}(\alpha^2)_{\text{LL}} - \mathcal{O}(\alpha^1)_{\text{LL}}\} \quad (1)$$

the SABSPV recipe is "multiplicative"

$$\mathcal{O}(\alpha^2)_{\text{LL}} \times \left\{ 1 + \frac{\mathcal{O}(\alpha^1) - \mathcal{O}(\alpha^1)_{\text{LL}}}{\text{Born}} \right\}. \quad (2)$$

³A table of the absolute cross sections is given in Ref. [2].

In both cases the $\mathcal{O}(\alpha^3)_{LL}$ can be easily used instead of $\mathcal{O}(\alpha^2)_{LL}$. Finally, in Fig. 1 we also show cross section denoted SABSPV2 (dots) which is obtained according to multiplicative prescription of SABSPV but using the cross sections from OLDBIS and LUMLOG⁴.

As we see in Fig. 1, there are distinct regularities in the results. The OLDBIS + LUMLOG additive ansatz coincides extremely well with the results of BHAGEN95, because the latter is also based on the additive recipe. The difference between BHLUMI and the result of additive recipes is, consistently with the older papers [14, 1], within 0.15% for the experimentally interesting range $0.25 < z_{\min} < 0.75$. The result of the multiplicative prescription of SABSPV agrees with BHLUMI better; in fact, it stays for the same z_{\min} range within 0.1% “permille box”. This is an interesting result if we remember that the multiplicative prescription is a little better from the point of view of physics, since it can effectively account for the part of the phase space with one real photon collinear and one real photon acollinear to e^\pm . The additive prescription simply ignores such configurations. Another encouraging result, albeit only for the unrealistic BARE1 ES, is the good agreement of the NLLBHA result with BHLUMI. If taken seriously it would mean that the missing $\mathcal{O}(\alpha^2)$ next-to-leading-log (NLL) contribution in BHLUMI is indeed below 0.1%. One would really need a similar result for more realistic ES’s. The above comparisons were also extended to the asymmetric version of the SES’s and they were also repeated for the situation when the Z-exchange and vacuum polarization were restored. As can be seen in Ref. [2], similar agreements were obtained. For LEP2 the agreements are slightly worse but definitely better than 0.20%.

2.1. New theoretical error estimate

Following the above results the new estimate of the total theoretical error for the BHLUMI cross section was obtained. It is summarized in Ta-

⁴The slight difference between SABSPV and SABSPV2 is of pure technical origin. The SABSPV2 result is not included in Refs. [2, 3] – was obtained after the LEP2 workshop.

ble 3 with the various components of the theoretical error listed. The older results of Ref. [1] and the conservative projection of the theoretical error for LEP2 are also given. The main progress has been made regarding the missing photonic $\mathcal{O}(\alpha^2 L)$, which was reduced by 30%. This result is based on the agreement between BHLUMI and SABSPV for all four standard event selections, see example in Fig. 1. The agreement within 0.1% between BHLUMI and NLLBHA for the unrealistic BARE1 trigger was also taken into account. The estimate of the missing photonic $\mathcal{O}(\alpha^3 L^3)$ contribution is based on the result from the new calculation embodied in the LUMLOG event generator (sub-generator in BHLUMI) [7]. As we see, the older value of this contribution was underestimated. The light pair contribution was also previously underestimated and as a result of work and discussion within the working group it went up to 0.03%. There is a condition attached to the use of 0.03% for pairs – one has to use at least the LL calculation of the light pair production effect. If not, then 0.04% is recommended. The new, slightly better value of the vacuum polarization error is the result of recent works [15] and [16]. The corresponding programs for vacuum polarization are included in BHLUMI [7]. The improved calculation with a new smaller error tag for the Z exchange was done during the workshop and published in Ref. [17]. The corresponding improvement of the matrix element for the Z exchange is implemented in BHLUMI [7].

The highest priority is now to calculate $\mathcal{O}(\alpha^2 L)$ contributions and to implement them in the BHLUMI Monte Carlo event generator. When this correction is under complete control, then the remaining biggest problem will be to determine and increase the technical precision of the Monte Carlo calculation. (At present, technical precision is combined with the missing $\mathcal{O}(\alpha^2 L)$ contribution.)

There is an ongoing effort to calculate complete $\mathcal{O}(\alpha^2 L)$ corrections for the SABH process, see for instance Refs. [12, 18] and the contribution of L. Trentadue in these Proceedings. In the next section, we shall present the new unpublished numerical result for some important $\mathcal{O}(\alpha^2 L)$ contribution, coming from the Cracow-

Type of correction/error	LEP1		LEP2
	Ref. [1]	Present	Present
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.20%
(a) Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015%	0.03%
(c) Vacuum polarization	0.05%	0.04%	0.10%
(d) Light pairs	0.01%	0.03%	0.05%
(e) Z-exchange	0.03%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

Table 3

Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within 1° – 3° , and for LEP2 it covers energies up to 176 GeV, and angular range within 1° – 3° and 3° – 6° (see the text for further comments).

Knoxville group.

Finally, let us mention only briefly the main results of the Bhabha Working group on the LABH process. In the working group the first systematic comparison of seven different calculations was done. The agreement of 0.5% close to the Z-peak was reached and at LEP2 agreement at the level of 2% was seen. Most of the programs were of the Monte Carlo type and the comparison was made for two realistic ES's, calorimetric and non-calorimetric, each for two values of the collinearity cut. The resulting precision estimate essentially confirmed the expectation. The important achievement was that it was the first systematic comparison of the LABH programs for a wide range of event selections. The result of the comparison is encouraging, but more work is obviously required if the theoretical error is to be reduced. We refer the reader for more details to the section on the LABH process in Ref. [2].

2.2. New results in $\mathcal{O}(\alpha^2 L)$

The calculation of the second-order QED matrix for the SABH process includes a calculation of the (a) two-loop second-order form factor, (b) one-loop correction to single photon emission process and (c) tree-level double-photon bremsstrahlung. The calculations of (a) and (c) exist in the literature and the fully differential distribution for (b) was not available until recently. In the following we describe numerical results obtained using the recently published results of Ref. [19] on the one-loop corrections to single-photon emission. The Feynman diagrams

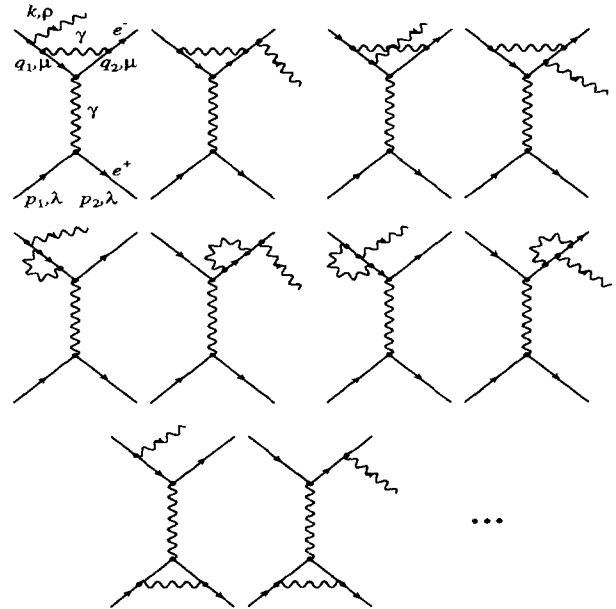


Figure 2. $\mathcal{O}(\alpha^2)$ single bremsstrahlung correction in $e^+e^- \rightarrow e^+e^-$ at low angles. Only the upper line real emission graphs are shown.

involved in this process are shown in Fig. 2. In the BHLUMI event generator this matrix element is implemented in the $\mathcal{O}(\alpha^2 L^2)$ approximation. The sub-leading contributions of $\mathcal{O}(\alpha^2 L)$ are incomplete in BHLUMI. Since the result of Ref. [19] is in principle *exact*, we therefore have a chance to check how large is, in fact, the missing $\mathcal{O}(\alpha^2 L)$

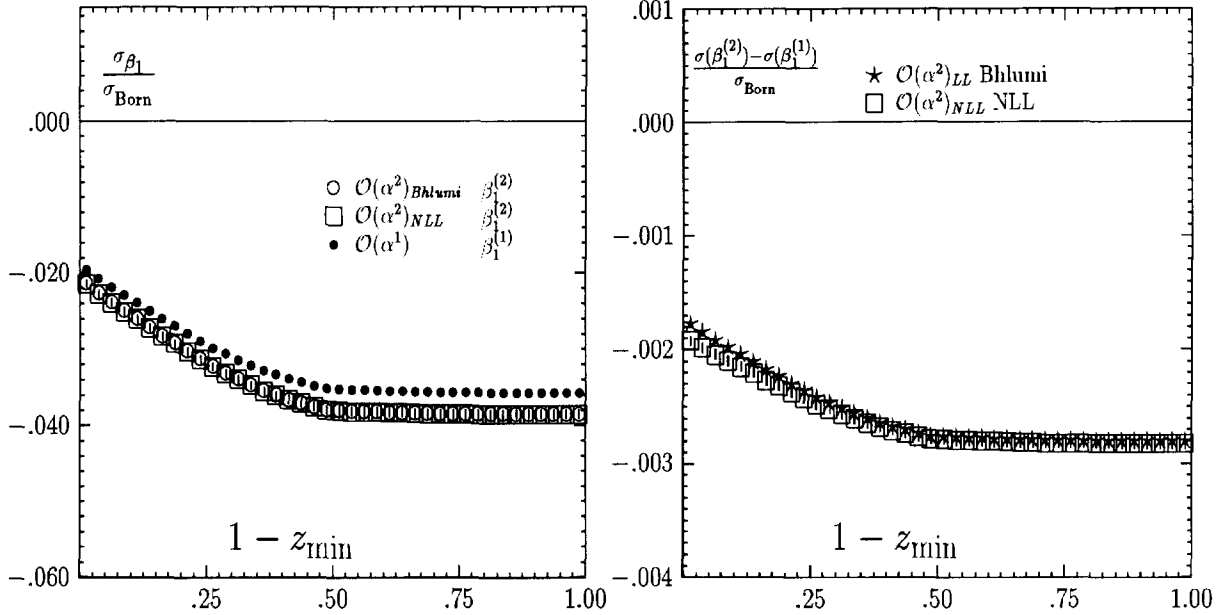


Figure 3. Monte Carlo result for entire $\bar{\beta}_1^{(r)}$, $r = 1, 2$ and for the pure second-order contribution $\bar{\beta}_1^{(2)} - \bar{\beta}_1^{(1)}$ for the SICAL Wide-Narrow event selection. Mass-terms are not included. All result divided by Narrow-Narrow Born cross section. Energy cut variable z_{min} is defined in Fig. 2 of Ref. [1].

part. We expect it generally to be 0.1% or less. There is, however, a certain problem from the start. How can we calculate something meaningful numerically from the formulas of Refs. [19] and [1] if the matrix element from the Feynman diagrams in Fig. 2 taken alone is infrared-divergent. We must combine it with double bremsstrahlung and the second-order part of the form-factor in order to get a finite cross section. The solution that we shall apply for the moment is to subtract both virtual and real singularities according to the work of Yennie, Frautschi, and Suura [20]. The infrared-divergent terms are trivial and uninteresting – they are proportional to the perfectly known first-order matrix element. Only the piece left after the subtraction contains non-trivial $\mathcal{O}(\alpha^2)$ corrections; it is finite and small. It is the so-called $\bar{\beta}_1$ function. The $\bar{\beta}_1$ function is a standard object in the YFS inclusive exponentiation scheme – it is already used in the BHLUMI

matrix element. It is defined as follows

$$\bar{\beta}_1^{(2)}(\mathbf{k}_i) = \left\{ D_{[1]}^{(2)}(\mathbf{k}) \exp(-2\alpha\mathcal{R}B) \right\} \Big|_{\mathcal{O}(\alpha^2)} - \tilde{S}_p(\mathbf{k}) \bar{\beta}_0^{(1)}, \quad (3)$$

where $D_{[1]}^{(2)}$ is the $\mathcal{O}(\alpha^2)$ squared matrix element for single-real-photon emission corresponding to diagrams in Fig. 2; the definitions of the infrared virtual formfactor B and of the real soft factor \tilde{S} are exactly the same as in Refs. [20, 1]. We are really interested in the pure $\mathcal{O}(\alpha^2)$ part of $\bar{\beta}_1^{(2)}$; we will therefore need to subtract the trivial first-order version of it defined as

$$\bar{\beta}_1^{(1)}(\mathbf{k}_i) = D_{[1]}^{(1)}(\mathbf{k}) - \tilde{S}_p(\mathbf{k}) \bar{\beta}_0^{(0)}. \quad (4)$$

The 3-body phase-space integration is done using the BHLUMI Monte Carlo; the results are shown in Fig. 3 for the realistic caloric ES called SICAL defined in Ref. [1] as a function of the energy cut z_{min} , also defined there. As we see in the

figure the difference between $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^1)$ is about 3% and therefore compatible with the generic size of the $\mathcal{O}(\alpha^2 L^2)$, see Table 2. The difference between the BHLUMI LL ansatz of Ref. [1]⁵ and the exact result of Ref. [19] is up to 0.015%. This must be multiplied by factor 2 because only the emission from the upper line is taken into account. We therefore obtain the total missing $\mathcal{O}(\alpha^2 L)$ to be 0.03%, i.e. well compatible with expectations. In the above results, the so-called mass-terms are still not included⁶. They are still being tested numerically. Note that this result was not known at the time of the LEP2 workshop and was not taken into account in Ref. [2] and Table 3. The above result shows the most complicated and difficult component of the $\mathcal{O}(\alpha^2)$ calculation, but it is still incomplete because the two-loop form-factor and double-bremsstrahlung contributions must be included. Once all three components are in BHLUMI, we shall be able to make the theoretical precision of the luminosity measurement better still. We hope that this will happen soon. Let us also express our hope that we shall be able to compare our results with those of Refs. [12, 18].

3. Summary

We have reported on the recent improvements on the precision of the QED calculations of the Bhabha process down to the 0.11% level. In particular we have shown recent results on the second-order exact calculation for one real and one virtual photon, which suggests that missing $\mathcal{O}(\alpha^2)$ contributions are below 0.1%. This opens the way to even further improvements in the theoretical error on the luminosity measurement at LEP.

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⁵In the presented results we used BHLUMI 4.03 with the corrected matrix.

⁶Their inclusion should not spoil the agreement – mass-terms are expected to be negligible.

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