

Mass of the H dibaryon in a chiral model

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 (Received 22 March 1985)*

We evaluate the mass of the dibaryon in a chiral model which includes the ω vector meson. We compare our results with those of the Skyrme model.

Calculations based on the MIT bag model¹ predicted the existence of a six-quark bound state called the H , with $J^\pi = 0^+$, $I = 0$, $Y = 0$, and a mass $M_H = 2150$ MeV, which is 80 MeV below the threshold for strong decay into $\Lambda\Lambda$. This prediction spurred some experimental searches which have been so far inconclusive. In the first,² the H was not seen, but the upper limit on its production cross section was too large to rule out its existence. A recent experiment³ claims to have seen one event which may be the H .

The H has been recently rediscovered as a soliton of baryon number $B = 2$ in the Skyrme model,⁴ and various estimates of its mass have been given.^{5,6} They all come out much lower than the bag-model prediction. Since the mass of the H is crucial in determining its stability with respect to strong decay into baryons, and therefore the details of its production and decay, it is important to ask how dependent the mass is on the particular choice of a chiral model which contains the baryons as solitons. For this reason, we compute the mass of the H in another model, which contains the ω meson but omits the Skyrme term.⁷ The two-flavor version of this model gave slightly better results for the static properties of nucleons than the Skyrme model did. (Both models are typically accurate to within 30%.)

The model we use is based on the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)(\partial^\mu\omega^\nu - \partial^\nu\omega^\mu) + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \beta\omega_\mu B^\mu + \frac{1}{16}F_\pi^2 \text{tr}(\partial_\mu U \partial^\mu U^\dagger), \tag{1}$$

where ω is the omega-meson field, m_ω is its mass, F_π is the pion decay constant, U is in $SU(3)$, and B^μ is the baryon current

$$B^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} \text{tr}[(U^\dagger \partial_\nu U)(U^\dagger \partial_\alpha U)(U^\dagger \partial_\beta U)]. \tag{2}$$

The coupling $\beta\omega_\mu B^\mu$ stabilizes the soliton solutions. Experimentally, $F_\pi = 186$ MeV and $\beta \leq 25.4$ (Ref. 7). The Lagrangian has exact chiral symmetry. The effect of explicit breaking will be discussed later.

The ansatz for the H is⁴

$$U_H(\mathbf{x}) = \exp[i\hat{\mathbf{x}} \cdot \Lambda f(r) + i[(\hat{\mathbf{x}} \cdot \Lambda)^2 - \frac{2}{3}]g(r)], \tag{3}$$

where $\Lambda_1 = \lambda_7$, $\Lambda_2 = -\lambda_5$, and $\Lambda_3 = \lambda_2$ generate an $SO(3)$ subgroup of $SU(3)$, $f(r) = g(r) = \pi$ at $r = 0$, and f and g vanish for $r \rightarrow \infty$. (See Fig. 1.) The ground state of the H is a spin singlet and an $SU(3)$ singlet, so it is described by a static soliton whose mass is purely classical. Substituting the ansatz (3) into (2) gives

$$M_H = 4\pi \int_0^\infty r^2 dr \left\{ -\frac{1}{2} \left(\frac{\partial\omega_0}{\partial r} \right)^2 - \frac{1}{2} m_\omega^2 \omega_0^2 - \beta\omega_0 B^0 + \frac{F_\pi^2}{8} \left[\left(\frac{\partial f}{\partial r} \right)^2 + \left(\frac{\partial g}{\partial r} \right)^2 + \frac{4}{r^2} (1 - \cos f \cos g) \right] \right\}. \tag{4}$$

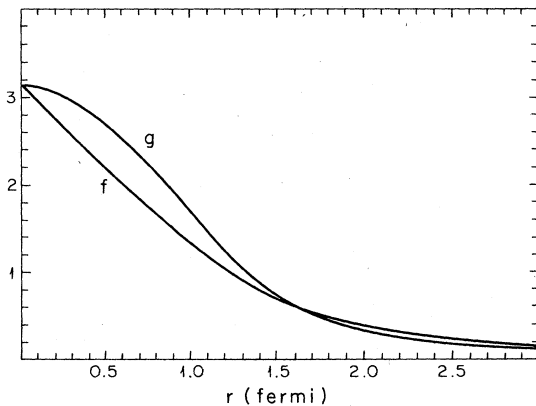


FIG. 1. The functions f and g which appear in the dibaryon ansatz for $F_\pi = 124$ MeV and $\beta = 15.6$.

The spatial components of ω_μ and B^μ vanish for a static soliton.⁷ The ω_0 field vanishes as $r \rightarrow \infty$, and $\partial\omega_0/\partial r = 0$ at $r = 0$. (See Fig. 2.) The baryon-number density obtained by substituting (3) into (2) is

$$B^0 = \frac{-1}{2\pi^2 r^2} \left[\frac{\partial f}{\partial r} (1 - \cos f \cos g) + \frac{\partial g}{\partial r} \sin f \sin g \right]. \tag{5}$$

This gives a total baryon number $B = 2$.

In order to compute M_H , we need to know F_π and β . A successful procedure in the two-flavor case has been to predict them by fitting the nucleon and Δ masses.⁷ In the chiral- $SU(2)$ case this procedure gave $F_\pi = 129$ MeV and $\beta = 15$. If we borrowed these values of F_π and β to compute the mass of the H , we would get $M_H = 1.68$ GeV. (In the Skyrme model the corresponding result is⁵ $M_H = 1.65$ GeV.) As no $SU(3)$ breaking has been considered here, this would correspond to a Λ mass $M_\Lambda = M_N = 938$ MeV, and so to an H binding energy of about 200 MeV.

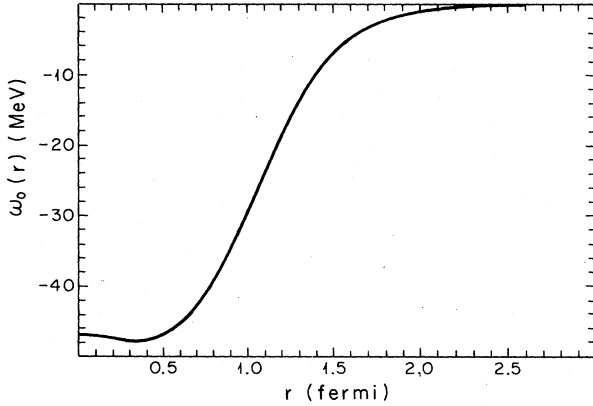


FIG. 2. The omega field ω_0 for $F_\pi = 124$ MeV and $\beta = 15.6$.

However, to use the SU(2) values for F_π and β is not, of course, the correct thing to do. One should compute F_π and β by fitting the $B=1$ quantized soliton masses to the observed octet and decuplet masses directly in the SU(3) \times SU(3) model. The SU(3) ansatz for the $B=1$ sec-

$$I = \frac{2\pi}{3} F_\pi^2 \int_0^\infty r^2 \sin^2 F dr + \frac{4\pi\beta^2}{3m_\omega^2} \int \int_0^\infty rr' dr dr' B^0(r) B^0(r') \times \{e^{-m_\omega(r+r')} [1 + m_\omega(r+r') + m_\omega^2 rr'] - e^{-m_\omega|r-r'|} (1 + m_\omega|r-r'| - m_\omega^2 rr')\} \quad (9)$$

is the moment of inertia, and

$$K = \frac{\pi}{2} F_\pi^2 \int_0^\infty r^2 (1 - \cos F) dr \quad (10)$$

The baryon-number density in (8) and (9) is⁷

$$B^0 = -\frac{1}{2\pi^2 r^2} \sin^2 F \frac{\partial F}{\partial r} \quad (11)$$

We have written (7) in a form that clearly separates the contribution of the strange quark. The first two terms are precisely those present in the two-flavor version of the model, and the third term is the result of adding the strange quark.

If we use (7) to fit $M_{B=1}$ to the octet ($J = \frac{1}{2}$, $C_2 = 3$) and decuplet ($J = \frac{3}{2}$, $C_2 = 6$) centers of mass, $M_8 = 1151$ MeV and $M_{10} = 1382$ MeV, we determine in the chiral limit $F_\pi = 87$ MeV and $\beta = 20$. Consequently we compute $M_H = 1.13$ GeV. The corresponding result in the Skyrme model is⁵ $M_H = 1.56$ GeV.

The reason why the estimates of the M_H with SU(2) and SU(3) parameters differ so much is due to the fact that this model overestimates the quantum contribution of the strange quark, which is the third term in (7). In fact, we find that this term alone contributes 43% of the total octet mass M_8 , while the SU(2) quantum term contributes only 5%. Because of such a large contribution from the strange quark, in order to fit $M_{B=1}$ to the physical baryon masses, we are forced to take a very low F_π , i.e., 33% below the SU(2) value. Consequently, the M_H comes out very low.

The fact that the collective coordinates of the strange

tor is

$$U_{B=1}(\mathbf{x}) = \begin{pmatrix} 0 \\ \exp[i\hat{\mathbf{x}} \cdot \boldsymbol{\tau} F(r)] & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (6)$$

where $F(r) = \pi$ at $r=0$ and $F(r) \rightarrow 0$ as $r \rightarrow \infty$. In Refs. 8 and 9 it was shown that quantization of (6) yields the correct quantum numbers for the octet and the decuplet. The soliton mass is

$$M_{B=1} = M_0 + \frac{1}{2I} J(J+1) + \frac{1}{2K} \left(C_2 - J(J+1) - \frac{N_c^2}{12} \right), \quad (7)$$

where J is the spin, C_2 is the SU(3) quadratic Casimir eigenvalue, and N_c is the number of colors. Above

$$M_0 = 4\pi \int_0^\infty r^2 dr \left\{ -\frac{1}{2} \left(\frac{\partial \omega_0}{\partial r} \right)^2 - \frac{1}{2} m_\omega^2 \omega_0^2 + \beta \omega_0 B^0 + \frac{F_\pi^2}{8} \left[\left(\frac{\partial F}{\partial r} \right)^2 + \frac{2}{r^2} \sin^2 F \right] \right\} \quad (8)$$

is the classical mass,

quark contribute so much to the energy is not actually totally surprising. We know that the static mass M_0 is order N_c and the rotational energy $J(J+1)/2I$ is order $1/N_c$ (Ref. 11). The contribution of the strange quark, although in this approach it comes totally from the collective coordinates, is instead of order 1, as one would expect from large- N_c QCD considerations. The reason for this is that both K and $C_2 - J(J+1) - N_c^2/12$ in (7) are order N_c , so that their ratio is order 1 (Ref. 12). Therefore the only thing we could reasonably expect is M_0 to be N_c times larger than the strange-quark contribution. But N_c is only 3, and it is simply an unfortunate accident of this model that the quantum correction due to the strange quark turns out to be as large as the classical mass. In the Skyrme model there is a similar problem, but the overestimate of the strange-quark contribution seems less severe.^{5,6,10}

Introducing a mass term in the Lagrangian to break chiral and SU(3) symmetry does not improve matters. In the Skyrme model F_π drops down to only 62 MeV, and consequently the mass of the H lowers to 1030 MeV (Ref. 6). Unfortunately, in our model the situation is even worse, since we are unable to find any F_π and β to fit the masses M_8 and M_{10} . The problem seems to be that the model overestimates the contribution of the strange-quark mass term, too.

In conclusion, we encounter in this model more severe problems than those already pointed out in the Skyrme model. Consequently, we are not able to make any definite assessment about the stability of the H . Maybe a pragmatic, although nonrigorous approach is to ignore altogether the contribution of the strange quark, and use the SU(2) parameters mentioned above, which give the H a binding

energy of 200 MeV. The justification for this might be found if there is a more realistic chiral model. A possibility is that introducing strange quarks on the same footing as up and down quarks is probably not the best approach, but rather they need to be introduced in chiral models in an alternative way. Progress in this direction has been made recently with a model which couples the kaon to an SU(2) chiral Lagrangian, thus breaking SU(3) from the start.¹³ In

such a model the mass of the H could possibly be evaluated more reliably.

We thank I. Klebanov and V. Kaplunovsky for valuable discussions. This material is based upon work supported under a National Science Foundation Graduate Fellowship. The research of C.R.N. was supported in part by National Science Foundation Grant No. PHY80-19754.

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