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Virtual Corrections to Bremsstrahlung in High-Energy Collider Physics: LHC and e^+e^- Colliders*

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We describe radiative corrections to bremsstrahlung and their application to high energy collider physics, focusing on the applications to luminosity measurement, fermion pair production and radiative return. We review the status of one loop radiative corrections in BHLUMI and the $\mathcal{KK}$MC, including cross checks with newer results developed independently for radiative return. We outline a YFS-exponentiated approach to the Drell-Yan process for LHC physics, including a discussion of the relevant radiative corrections.

1. RADIATIVE CORRECTIONS TO BHABHA SCATTERING

The BHLUMI Monte Carlo (MC) program[1] was developed as a precision tool for calculating the small-angle Bhabha luminosity process at SLC and LEP, and with continued development, it will continue to be a valuable tool meeting the requirements of a next-generation linear e^+e^- collider, such as the proposed ILC. Central to this program's success was an exact treatment of the phase space for n photon bremsstrahlung. A YFS-exponentiation[2] procedure allows all IR singularities to be canceled exactly between real and virtual emission processes to all orders. The leading soft photon effects are exponentiated, and IR-finite YFS residuals are then calculated exactly to the order required to reach the desired precision level.

BHLUMI attained a total error budget of 0.061% for LEP1 parameters and 0.122% for LEP2 parameters for a typical calorimetric detector scenario.[3] To assure this precision level, it was necessary to calculate the most important unimplemented effect in BHLUMI4.04, which was the next to leading-log (NLL) contribution to the two-photon radiative corrections. The double real photon and single real plus virtual photon corrections to the small-angle Bhabha scattering process were calculated exactly in refs. [4,5] When added to the known two-loop virtual correction[6], these results showed that the $\mathcal{O}(\alpha^2 L)$ corrections enter at the 0.027% level for LEP1 parameters and 0.04% level for LEP2 parameters. Here, $L = \ln(|t|/m_e^2)$ is the "large logarithm" entering into a leading log expansion.

Implementing these exact $\mathcal{O}(\alpha^2)$ results in BHLUMI would eliminate these contributions to the error budget. The only remaining unimplemented $\mathcal{O}(\alpha^2)$ radiative corrections would then be up-down interference effects in which two virtual photons are exchanged between the e^+ and e^- line, which are suppressed at small angles, and nominally enter at the level of 0.004% or less for angles below 9°.[7,8] These contributions, represented by diagrams of the type shown in Fig. 1, could thus be safely neglected for SLC and LEP physics.

For ILC physics, where the goal is to reach 0.01% in the small-angle Bhabha luminosity process, it is desirable to carefully check the magnitude of the up-down interference terms, and to implement them if they turn out to be significant. A key ingredient in the comparison, the five-point box integral appearing in the second and fourth diagram in Fig. 1, has recently been provided by the Looptools 2.2 package.[9] A num-

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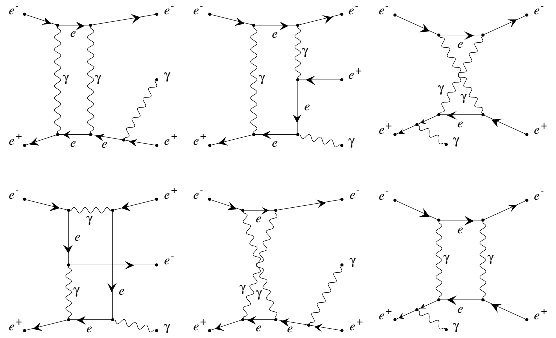


Figure 1. One-loop up-down interference diagrams for t-channel positron line emission. The internal photons may also be replaced by a Z boson.

ber of other $\mathcal{O}(\alpha^2)$ calculations which have appeared recently[10,11] should also provide valuable insight into effects which may need to be implemented in the small-angle Bhabha calculation to reach ILC precision specifications.

2. RADIATIVE CORRECTIONS TO FERMION PAIR PRODUCTION

Fermion pair production plays a critical role in extracting precision electroweak physics from e^+e^- colliders. This process is calculated by another YFS-exponentiated MC program, the $\mathcal{KK}$MC.[12] Again, the photonic radiative corrections play an essential role in calculating the YFS residuals through order α^2. These have been calculated exactly, including finite-mass corrections, for initial state and final state radiation.[13]

Using helicity spinor techniques,[14,15] a concise and stable representation for the $\mathcal{O}(\alpha^2)$ initial or final state radiation amplitude has been obtained, including finite-mass corrections. The matrix element for hard photon initial-state emission with one virtual photon may be expressed as

$$\mathcal{M}_1^{\mathrm{ISR}(1)} = \frac{\alpha}{4\pi}\mathcal{M}_1^{\mathrm{ISR}(0)}(f_0 + f_1 I_1 + f_2 I_2), \qquad (1)$$

where $\mathcal{M}_1^{\mathrm{ISR}(0)}$ is the tree-level matrix element for single hard photon emission, f_i are scalar form factors and I_i are spinor factors defined in ref. [13].

The single hard photon cross section is of particular interest in radiative return applications[16–18], where initial state radiation is used to reduce the effective beam energy, allowing a fixed energy machine to probe a range of energies. A MC program PHOKHARA was developed to calculate radiative return at Φ and B factories.[19,20] The same radiative corrections are relevant for a high-energy e^+e^- collider investigating physics around the Z peak, for example. It is therefore useful to compare the radiative corrections obtained for both the $\mathcal{KK}$MC and PHOKHARA in detail. Both calculations claim the same level of exactness, including the same diagrams as well as electron mass corrections relevant for collinear bremsstrahlung.

We have compared the virtual corrections to initial state hard-photon emission calculated in ref. [21,22] (KR) for PHOKHARA to those calculated in ref. [13] (JMWY) for the $\mathcal{KK}$MC in the case of muon pair production. Analytically, it was found that in the absence of mass corrections, both expressions agree to NLL order ($\mathcal{O}(\alpha^2 L)$ in the integrated cross section).[23] A compact expression for NLL limit of the matrix element was obtained in ref. [13], where it was shown that the terms f_1 and f_2 in eqn. (1) vanish to NLL order, and the helicity-averaged NLL limit of f_0 is

$$\begin{aligned}
\mathrm{Re}\,\langle f_0^{\mathrm{NLL}}\rangle &= 2\pi \mathrm{Re}\, B_{\mathrm{YFS}}(s) + L - 1 \\
&+ 2\ln r_1 \ln(1-r_2) - \ln^2(1-r_1) \\
&+ 3\ln(1-r_1) + \frac{r_1(1-r_1)}{1+(1-r_1)^2} \\
&+ 2\mathrm{Sp}\,(r_1) + (r_1 \to r_2). \qquad (2)
\end{aligned}$$

where $L = \ln(s/m_e^2)$ is the “large logarithm” in the leading log expansion, $r_i = 2p_i \cdot k/s$ measures the inner product of one of the incoming fermion momenta p_i with the hard photon momentum k, $\mathrm{Sp}(x) = \mathrm{Li}_2(x)$ is the dilogarithm (Spence) function, and

$$\begin{aligned}
4\pi \mathrm{Re}\, B_{\mathrm{YFS}}(s) &= \left(2\ln\frac{m_\gamma^2}{m_e^2} + 1\right)(L-1) \\
&- L^2 - 1 + \frac{4\pi^2}{3} \qquad (3)
\end{aligned}$$

is the IR-divergent virtual YFS form factor.

Since the two results are known to agree with the NLL limit calculated using eqn. (2), the NLL limit is subtracted in each case, permitting the NNLL contributions and collinear mass corrections to be investigated in the context of the $\mathcal{KK}$MC. Fig. 2 shows the results of a $\mathcal{KK}$MC run calculating the NNLL contribution to muon pair production at a CMS energy of 500 GeV for 10^8 events, both with and without the mass corrections. The cross-section is integrated up to a radiated photon energy fraction of $v_{\max}$ (with $v = r_1 + r_2$) using the YFS residual $\beta_1^{(2)}$ for one hard photon at $\mathcal{O}(\alpha^2)$, subtracting the NLL contribution obtained using eqn. (2). The result is normalized with respect to the non-radiative Born cross section for muon pair production.

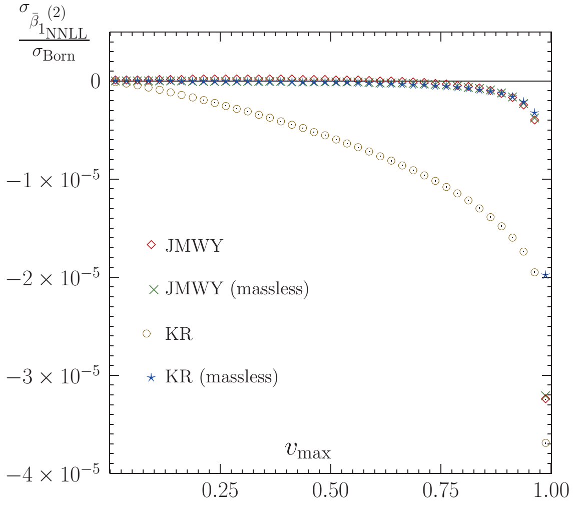


Figure 2. Comparison of NNLL contributions to the virtual correction to the single hard photon cross section for muon pair production at a CMS energy of 500 GeV.

It is found that the maximum difference between the complete KR and JMWY results (from the next-to-last bin) is 1.6×10^{-5} units of the Born cross section. Most of this apparently comes from differences in the treatment of the mass corrections. KR uses an expansion in $m_e^2/r_i s$, while JMWY uses a technique developed by Berends *et al.*[24] for adding the mass corrections required in collinear limits to a calculation obtained using massless spinors. Without mass corrections (comparing the massless points), the results agree to within a part per million. This agreement is better than noted previously[23,25–27] due to improvements in the stability of the algorithms used. Direct comparisons of the PHOKHARA and $\mathcal{KK}$MC programs have also been conducted.[28,29]

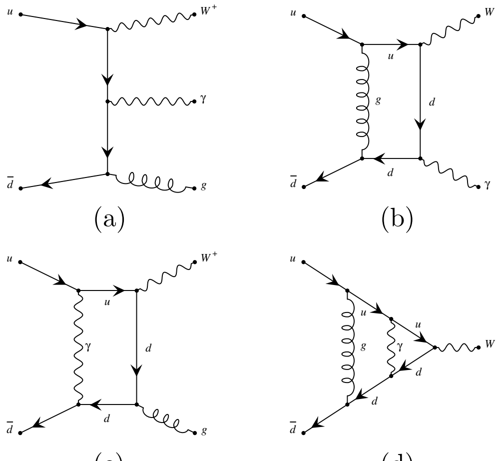
It is interesting to note that the size of the NNLL part of the corrections implemented by JMWY never exceed 4×10^{-6} up to $v_{\max} = 0.975$, and reach -3.25×10^{-5} in the last bin, where $v_{\max} = 0.9875$. This suggests that for most purposes, the considerably simpler NLL result represented by eqn. (2) will suffice.

3. THE DRELL-YAN PROCESS

The Drell-Yan process plays a role at hadron colliders which is as basic as the Bhabha scattering or pair production cross section at e^+e^- colliders. In fact, W and Z production has been proposed as the luminosity process for the LHC.[30] A fully exclusive calculation of the parton-level cross sections is needed at the $1 - 2\%$ level for upcoming LHC physics. These cross sections are currently known at the 10% level, using NLO matrix elements.

While NNLO results are available for the integrated cross section[31] and rapidity distribution[32], a fully-exclusive NNLO cross section needed for a MC event generator is not yet available. Moreover, electroweak radiative corrections will be required as well. Reaching the desired LHC precision will require corrections of order α_{s}^2 and order α_{ew}, including mixed $\mathcal{O}(\alpha_{\mathrm{s}}\alpha_{\mathrm{ew}})$ contributions. Examples of the latter diagrams are shown in Fig. 3.

The hard parton-level processes must be calculated and combined with PDFs in a MC program designed to generate the desired distribution of partons plus mixed QCD and QED bremsstrahlung. This will require a careful implementation of the multiple gluon and photon



(a) (b)

(c) (d)

Figure 3. Examples of mixed photonic and gluonic initial-state radiative corrections: (a) real photon + gluon emission, (b) real photon with QCD loop, (c) real gluon with electroweak loop, (d) 2 loops, mixed QCD/electroweak. The final state fermions from the W^+ decay are not shown.

phase space. Experience in the electroweak sector suggests that YFS exponentiation will provide a strong tool for implementing the multiple-emission phase space, giving very precise control over the soft and collinear limits.

The large numbers of diagrams creates a challenge for obtaining an expression that can be evaluated quickly enough for MC implementation, and evaluated in a numerically stable manner. Common reduction methods based on the Passarino-Veltman technique[34] can produce millions of terms, which are both slow to evaluate in a MC setting, and prone to numerical instabilities due to the large numbers of terms added and potential cancellations among them. Thus, a significant part of this problem will involve developing and testing new methods for organizing and calculating the terms in a stable manner.

Once the parton-level matrix element is obtained, it may be incorporated into a "QCED-Exponentiated" MC program, implementing a procedure similar to YFS exponentiation in a combined QED and QCD setting to construct the exact phase space for multiple gluon and photon radiation. This requires extending the YFS calculus to non-abelian gauge theory, with due care in handling the genuine non-abelian singularities which arise in QCD. The QCD and QED exponentiation will be conducted at order $\alpha_s^2 L$ on an event-by-event basis in the presence of showers without double-counting of soft and collinear emission effects. Further details of this construction will be presented elsewhere in these proceedings.[33]

4. SUMMARY AND OUTLOOK

A careful calculation of higher-order bremsstrahlung corrections led to a precision tool (BHLUMI) for Bhabha luminosity calculations. Incorporating the second order photonic corrections obtained to test BHLUMI's precision may be enough to reach the 0.01% level proposed for the ILC. Calculating the $\mathcal{O}(\alpha^2)$ up-down interference contribution will help to clarify this. A number of second-order Bhabha scattering results are now available which should be useful for testing BHLUMI's precision at this higher level.

We have also described cross-checks on the second-order photonic radiative corrections for fermion pair production developed for the $\mathcal{KK}$MC. Comparisons with similar initial-state radiative corrections developed for PHOKHARA show agreement on the order of 10^{-5} in units of the Born cross section at ILC energies. Remaining differences may be attributed primarily to differences in the handling of finite mass corrections in the collinear limits, and possible residual numerical instabilities for photons radiated near the fermion pair production threshold.

Finally, we have outlined a program for carrying the successes of the YFS-exponentiated MC framework developed for SLC and LEP physics into the hadronic realm of the LHC, where a precise calculation of the parton-level diagrams can be combined with a combined QCD and QED exponentiation framework, with complete control of the multiple photon and gluon phase space, to develop a MC for the Drell-Yan process which can reach the LHC precision requirements.

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