

# THEORETICAL CALCULATIONS OF THE BHABHA PROCESS

S. JADACH<sup>\*,a,b</sup>, M. MELLES<sup>c</sup>, W. PŁACZEK<sup>c</sup>, E. RICHTER-WAŚ<sup>d</sup>,  
M. SKRZYPEK<sup>a</sup>, B. F. L. WARD<sup>b,c,e</sup>, Z. WAŚ<sup>a</sup> and S.A. YOST<sup>c</sup>

<sup>a</sup>*Institute of Nuclear Physics, ul. Kawiory 26a, Kraków, Poland,*

<sup>b</sup>*CERN, Theory Division, CH-1211 Geneva 23, Switzerland,*

<sup>c</sup>*Department of Physics and Astronomy,*

*The University of Tennessee, Knoxville, Tennessee 37996-1200,*

<sup>d</sup>*Institute of Computer Science,*

*Jagellonian University, Kraków, Poland*

<sup>e</sup>*SLAC, Stanford University, Stanford, California 94309,*

*\*Presented by S. Jadach*

We present the current status of the theoretical high precision calculations of both small and large angle Bhabha scattering at high energies for realistic experimental set-ups. We report on the recent improvements in the precision of these calculations at both LEP1/SLC energies and at LEP2 energies. The best achieved precision in the theoretical calculations of the small angle Bhabha cross section for the luminosity is 0.11% and of the large angle Bhabha cross section is 0.3%.

## 1 Introduction

The Small Angle Bhabha (SABH) process below about  $6^\circ$  in the scattering angle is dominated by the gamma  $t$ -channel exchange and another one, the Large Angle Bhabha (LABH) process above  $6^\circ$ , gets dominant contributions from various  $s$ -channel (annihilation) exchanges. The SABH process is used almost exclusively to determine the luminosity in the  $e^+e^-$  colliders, using special luminometer sub-detectors. The LABH process is used for precision electroweak tests of the Standard Model (SM), in particular the electron partial width  $\Gamma_e$  of the Z boson. In this presentation we shall mainly elaborate on the SABH process. At LEP at  $\sqrt{s} = M_Z$ , in the  $1^\circ$ – $3^\circ$  angular range the SABH process gives about four times more events than Z decays. It is therefore very well suited for precise measurements of the luminosity from the point of view of the statistical error. It is dominated by “known physics”, that is by the  $t$ -channel exchange of a photon and is therefore calculable from the “first principles”, i.e. from the Lagrangian of the Quantum Electrodynamics (QED) with the standard Quantum Field Theory methods, Feynman diagrams, etc.

## 2 Small angle Bhabha

The calculation of the small angle Bhabha cross section for the luminosity calculation in the LEP

era was based the multiphoton Monte Carlo BHLUMI <sup>1,2</sup> from the beginning of the LEP data analysis. Note that at PEP/PETRA times calculation of the SABH cross section was based on work of Berends and Kleiss<sup>3</sup> and the corresponding Monte Carlo program with presumable precision of 1-2%. The precision tag of BHLUMI was set five years ago to be 0.5%<sup>4</sup> and was gradually improved to 0.25%<sup>5</sup>, then to 0.16%<sup>6</sup> and recently down to 0.11%. We shall report here on this recent step forward which was achieved within the recent LEP2 workshop<sup>7</sup>. Of course, the main reason for investing the effort in theoretical evaluation of the SABH cross section was the dramatic progress in the experimental error of the LEP luminometry. The typical LEP luminometer of the single LEP collaboration is now capable to measure the luminosity cross section with precision around or even below 0.1%<sup>8</sup> and, as pointed out by A. Blondel at these proceedings, the combined LEP experimental error is close to 0.05%. Obviously, theoretical calculations, in spite of big progress, are in terms of precision still behind the LEP experiments.

The present status of the theoretical error of the SABH process as calculated by BHLUMI 4.04<sup>16</sup> for the realistic SICAL-type event selection is summarized in table 1. This table was first presented in the proceedings of the LEP2 workshop<sup>7</sup> and published<sup>9</sup> afterwards. It is the result of a joint effort of the Bhabha Working Group

Type of correction/error	LEP1		LEP2
	Past <sup>6</sup>	Present	Present
(a) Missing photonic $\mathcal{O}(\alpha^2 L)$ <sup>8,9</sup>	0.15%	0.10%	0.20%
(b) Missing photonic $\mathcal{O}(\alpha^3 L^3)$ <sup>10</sup>	0.008%	0.015%	0.03%
(c) Vacuum polarization <sup>11,12</sup>	0.05%	0.04%	0.10%
(d) Light pairs <sup>13,14</sup>	0.01%	0.03%	0.05%
(e) Z-exchange <sup>15</sup>	0.03%	0.015%	0.0%
Total	0.16%	0.11%	0.25%

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for the angular range within  $1^\circ - 3^\circ$ , and for LEP2 it covers energies up to 176 GeV, and angular range within  $1^\circ - 3^\circ$  and  $3^\circ - 6^\circ$  (see the text for further comments).

of the 1995 workshop on LEP2 physics, notably of the authors of BHLUMI, SABSPV, BHAGEN95 and NLLBHA programs/calculations <sup>7</sup>.

The dominant QED bremsstrahlung uncertainty, see entry (a) in table 1, is based on the comparison of the programs BHLUMI, SABSPV, BHAGEN95 and NLLBHA. The example of such a comparison is shown in fig. 1 where the agreement between BHLUMI and SABSPV within 0.1% for any type of ES was instrumental in reaching this number. The agreement of NLLBHA and BHLUMI for the unrealistic BARE1 ES was taken also into account<sup>a</sup>. The so-called OLDBIS+LUMLOG<sub>h.o.</sub> calculation and BHAGEN95 are almost equivalent and they depart more than 0.1% from BHLUMI for understandable reasons<sup>7</sup>. On the basis of these comparisons, we have reduced the pure bremsstrahlung error on the calculation done by us in BHLUMI4.04 to 0.10% compared to our earlier estimate<sup>6</sup> of 0.15%, for example. The entries (b)-(e) in table 1 are also improved and are based on the new works which are referenced in the table.

Continuing in this way, using the results above and making further cross checks with our own semi-analytical results and with the available results from the literature as described in the LEP2 workshop proceedings <sup>7</sup>, we arrive finally<sup>9</sup> at the total theoretical precision budget shown in table 1, which also contains our previous theoretical precision results <sup>6</sup> and results for the LEP2 regime as indicated.

The key ingredient in the future improvements on the BHLUMI theoretical error beyond the sta-

<sup>a</sup>The NLLBHA calculation has in principle a more complete QED matrix element than BHLUMI but is limited to certain not very realistic ES's.

tus summarized in table 1 will be the upgrade of the QED matrix element to the level of the exact (or beyond leading-log order) $\mathcal{O}(\alpha^2)$ . In fig.2 we present the results of four of us (S.J., M.M., B.F.L.W., S.A.Y.) on the exact virtual one-loop correction to the hard bremsstrahlung process in small angle Bhabha scattering <sup>17,18</sup>. We show in this figure the comparison, for the NW ALEPH SICAL luminometer acceptance, of the pure second order contribution  $\bar{\beta}_1^{(2)} - \bar{\beta}_1^{(1)}$  to the hard photon residual  $\bar{\beta}_1$  (see Refs. <sup>19,1</sup> for the precise definition the  $\bar{\beta}_n$ ) as calculated by BHLUMI4.x and as calculated with the exact results <sup>18</sup>. (We stress that, prior to our work, no published, exact, completely differential result for this correction was available). We are currently implementing the analogous comparison to that in Fig. 2 for the massive case, which would then allow us to arrive at the final precision on the bremsstrahlung correction for BHLUMI4.x<sup>20</sup>. We should note that Arbuzov et.al. <sup>21</sup> have also recently obtained results for the next-leading-log contribution to the SABH process. We will compare our results with their final formulas elsewhere<sup>20</sup>.

### 3 Large angle Bhabha

In fig. 3 we give a glimpse on the progress made recently in the calculations of the large angle Bhabha (LABH) process. This figure is an improved version of the figure from the Bhabha Working Group of the LEP2 workshop <sup>7</sup>. We compare here our program BHWIDE <sup>22</sup> with those of BABAMC <sup>23</sup>, of the semi-analytical program TOPAZ0 <sup>24,25</sup>, of the MC event generator BHAGENE3 <sup>26,27</sup>, of the MC event generator BHAGENE95 <sup>28,29,30,31</sup>, and of the MC event generator

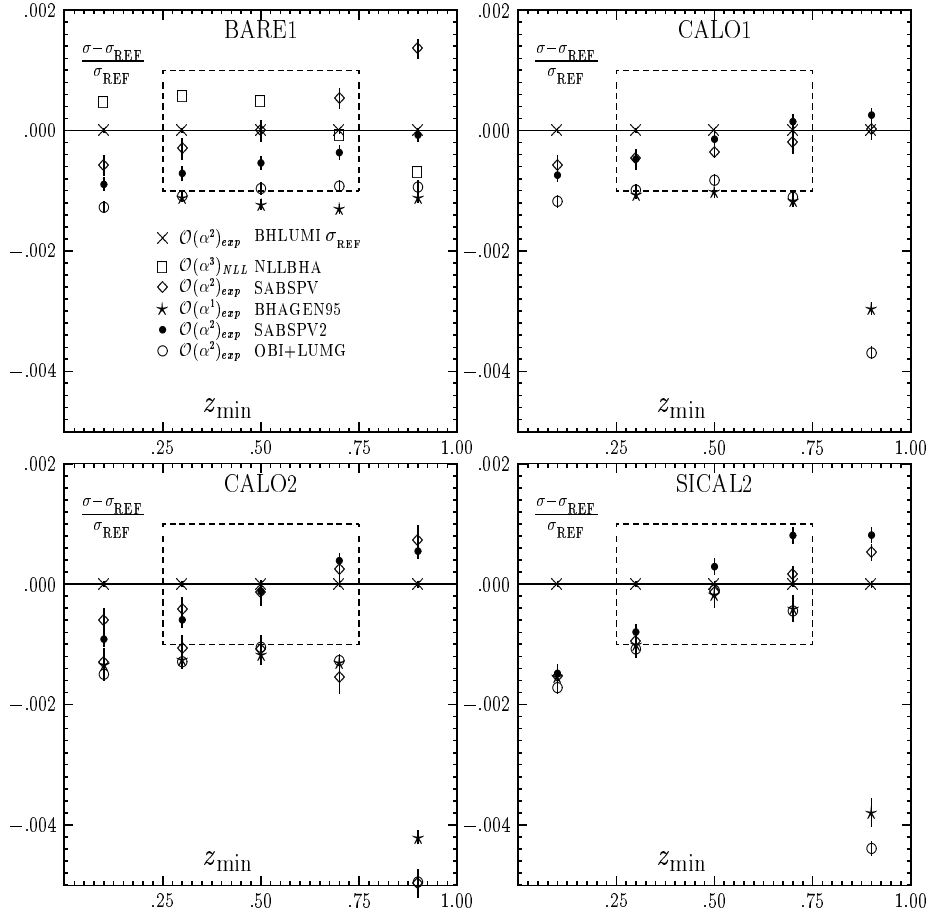


Figure 1: Monte Carlo results for the symmetric Wide-Wide ES's BARE1, CALO1, CALO2 and SICAL2, for matrix elements beyond first-order.  $Z$  exchange, up-down interference and vacuum polarization are switched off. The center of mass energy is  $\sqrt{s} = 92.3$  GeV. In the plot, the  $\mathcal{O}(\alpha^2)_{exp}^{YFS}$  cross section  $\sigma_{BHL}$  from BHLUMI 4.02.a is used as a reference cross section.

UNIBAB<sup>32b</sup>. This we do for the 8 CMS energy values: 88.45, 89.45, 90.20, 91.19, 91.30, 91.95, 93.00, 93.70 GeV, which are denoted in the figure as energy points 1, 2, ..., 8, respectively. Based on the above comparison and the other ones<sup>7,22</sup> we conclude that, for the CALO ES, within  $\pm 100$  MeV of the  $Z$  peak, the total precision of BHWIDE is 0.3% and off peak, within  $+2.5/-2.75$  GeV thereof, we set this precision at 0.5% in the LEP1 energy regime. This confirms and improves the earlier<sup>36</sup> precision estimated for the LABH process. Furthermore, the new calculations are available in the form of the Monte Carlo event generator<sup>22</sup>.

<sup>b</sup>In the rest of the comparisons<sup>7,22</sup> ALIBABA<sup>33,34</sup> and the Monte Carlo integrator program SABSPV<sup>35</sup> were also included.

## 4 Conclusions

In this work, we have presented the current status and outlook for the theoretical calculations for both SABH and LABH processes. For the SABH process, we showed that the total precision tag of 0.11% in version 4.04 of BHLUMI has been achieved and that all formulas needed for achieving the total precision tag  $\sim .05\%$  are now being implemented and cross-checked. For the LABH scattering at LEP1/SLC and LEP2 energies, we have a new exact  $\mathcal{O}(\alpha)$  YFS exponentiated MC BHWIDE 1.00. It features 0.5% precision in general at LEP1 (0.3% on the  $Z$  peak itself) and 1.5% at LEP2 energies, both state of the art results for MC event generators. Thus, in both the SABH and the LABH, we now have the state of the

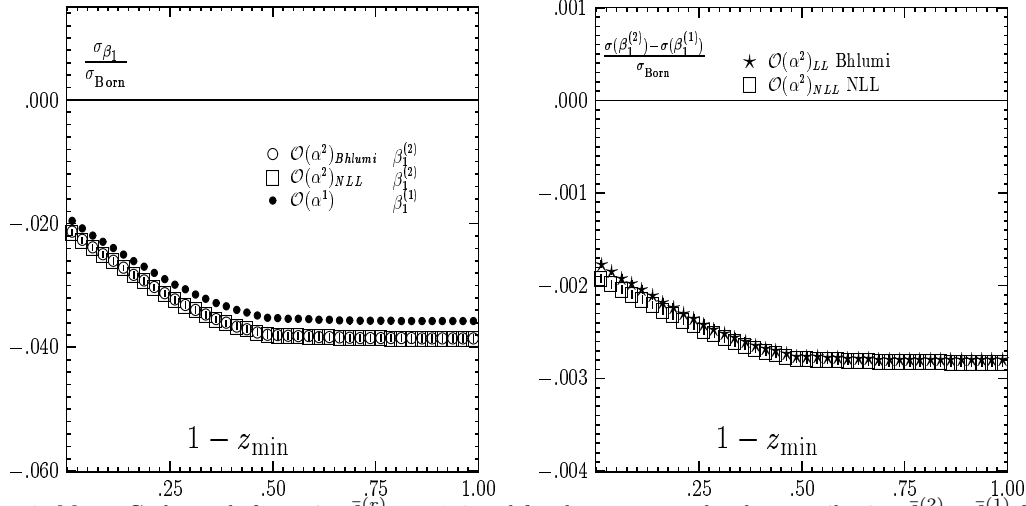


Figure 2: Monte Carlo result for entire  $\bar{\beta}_1^{(r)}$ ,  $r = 1, 2$  and for the pure second-order contribution  $\bar{\beta}_1^{(2)} - \bar{\beta}_1^{(1)}$  for the SICAL Wide-Narrow event selection. Mass-terms are not included. All result divided by Narrow-Narrow Born cross section. Energy cut variable  $z_{\text{min}}$  is defined in the literature.<sup>6</sup>

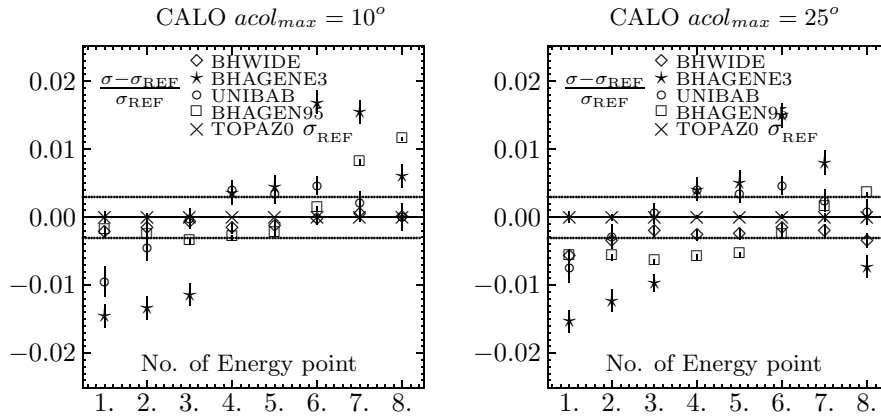


Figure 3: Monte Carlo results for CALO trigger, for two values ( $10^\circ$  and  $25^\circ$ ) of collinearity cut. Center of mass energies (in GeV) close to Z peak. In the plots cross section  $\sigma_{\text{REF}}$  from TOPAZO is used as a reference cross section. Cross sections in nb. Two horizontal dotted lines indicate the 0.3% band, for reference.

art calculations of the respective radiative corrections realized on an event-by-event basis so that arbitrary detector cuts are accessible. We find this situation exciting and we look forward to the many further applications of our results at the various  $e^+e^-$  colliding beam facilities such as LEP1, LEP2, SLC, SLAC B-Factory, BELLE, BES, etc.

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